ULTRA-SONIC OSCILLATIONS

PART I

U L T R A - S O N I C   O S C I L L A T I O N S

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A report of work carried out
for the Degree of Master of Science,
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Presented to the Committee on Graduate Studies of the

University of Alberta, April 1, 1922.

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EDMONTON.

Contents.

INTRODUCTION

The term "Ultra-Sonic Oscillations" is used to designate acoustic vibrations of very high pitch, above the limits of audibility. The waves in a fluid set up by these vibrations are short, ranging from about five or six centimetres to a fraction of a centimetre, and, as a result, it is possible to reflect, refract and diffract them by bodies of ordinary size.

A diffraction phenomenon, which underlies all work in ultra-sonics, takes place when a plate or diaphragm is set into high frequency longitudinal vibration. Mathematical calculations show that if all portions of the face of diaphragm oscillate in the same phase and if the diameter of the diaphragm (assumed circular) is large in comparison with a wave length, then nearly all the ultra-sonic energy is radiated in the form of a central beam (like a searchlight) whose angular width is proportional to the ratio of the ultra-sonic wave length radiated to the diameter of the diaphragm.

If ships are fitted with ultra-sonic transmitting and receiving devices, it is possible to signal along the ultra-sonic beam from one craft to another under the surface of the water, and this system of signalling is both secret and directive.

Mathematical calculations also show that when ultra-sonic waves in water strike an object whose density and elasticity are very different from those of water, the waves are reflected back along their path and the reflected beam may be detected as an "echo" at the source of the waves. In this way the presence of submerged objects like icebergs, rocks, hulls of ships, etc., can easily be detected at a distance of two miles or more and darkness or fog are of no consequence. The distance of the reflecting object can easily

be estimated from the length of time required for the travel and return of the transmitted signal, and an accurate bearing can also be given. The development of this branch of the subject should prove of immense value to navigation.

The present investigation is confined to a study of the scientific principles underlying the formation and propagation of ultra-sonic waves and of the ultra-sonic beam but no attempt is made here to apply these principles.

One of the chief points of scientific interest is that the wave lengths under ~~of ordinary~~ consideration are necessarily intermediate between the long wave lengths of ordinary sound and the extremely short wave lengths of light. By using ultra-sonic oscillations, a wave length of one or two centimetres is readily obtained and apparatus with diamter of the "aperture" of the order of a few wave lengths, or a fraction of a wave length, can easily be designed. This makes it possible to investigate experimentally such new problems in wave physics as the following: interference and diffraction phenomena within a few wave lengths of the "aperture"; the effect of the ratio between the wave length and the thickness of a reflecting or refracting partition upon the proportion of energy reflected and transmitted; the distribution of energy reflected, refracted or diffracted by a body whose linear dimensions are of the order of a few wave lengths; and many other similar problems.

ULTRA-SONIC OSCILLATIONS

By a mathematical investigation, E. Verdet (') has shown that if a parallel beam of light is passed through a small circular opening in an opaque screen, in a direction perpendicular to the screen, an interference phenomenon takes place and alternate zones of reinforcement and neutralization occur. The central zone is in the form of a beam whose angle of divergence depends upon the diameter of the aperture and the wave length of the energy passing through it. In this central zone the intensity of illumination is far greater than in any surrounding zones. Verdet has developed an expression to show the effect of the diameter of the opening, and of the wave length of the waves passing through it, upon the position of the reinforcement and neutralization zones. He also gives the relative maximum intensities in the different reinforcement zones. An outline of Verdet's work is given below in the section on the "Shape of the Ultra-Sonic Beam." The problem arose to make practicable and available for practical use the analagous phenomenon ~~arose~~ in the case of sonic or of ultra-sonic waves.

G. B. Airy (") has also shown that when light passes through a hole, alternative zones of reinforcement and neutralization occur. He points out that in the case of ordinary or audio-frequency, sound waves passing through the hole this interference phenomenon would not occur and the energy would be distributed uniformly in all directions. Airy shows that the difference between the optical and sonic conditions is due to the relation between the size of the hole and the wave lengths involved.

If R represents the radius of a circular hole through which plane waves are passing in a direction normal to the plane of the hole, and λ is the wave

(') E. Verdet: Lecons d'Optique Physique. Tome 1, page 301.

(") G.B. Airy: Mathematical Tracts,- "Intensity of Light Passing Through a Hôle".

length of these waves, then when λ is very much greater than R (the case of ordinary sound waves) the energy is distributed, spherically, uniformly and from the hole as centre. But if λ is much smaller than R (the case of a pin hole and waves of light) reinforcement and neutralization zones are formed. From Airy's work it would appear that if sound waves of sufficiently small wave length could be obtained, the same interference phenomenon would occur as occurs in optics, and such has been shown to be the case qualitatively with waves from high pitch bird calls in air and ripple waves in water('), though no precise and accurate measurements have been made on the phenomena.

The criterion for obtaining diffraction effects in all these cases, is that the diameter of the hole must be large in comparison with the wave length of the waves, and that all points in the plane of the hole are in the same phase of vibration. This will be the case if the waves, on reaching the hole, in a direction perpendicular to the plane of the hole, are plane. Virtually the same conditions prevail if, instead of plane waves passing through a hole, we have a solid plate with parallel faces set into vibration in such a way that all portions of its face are vibrating in the same phase. If also the diameter of this plate (assumed circular) is large in comparison with a wave length, the plate, when immersed in an elastic medium, should emit a radiation distributed in zones of reinforcement and interference.

The phenomenon has in recent years been realized in practice. A plate was set oscillating with all portions of its face in the same phase(") and served the same purpose, from the point of view of the interference theory, as did the aperture in the optical case. In the investigations the diameter of the oscillating plates have varied from about fifteen centimetres to forty-five centimetres and therefore ultra-sonic wave lengths of a few millimetres to a few

(') Lord Rayleigh: Proc. Royal Inst. Jan 20, 1888, Phil. Mag. Vol. IX, p. 281 1890.
(") see section 2,--" Ultra-sonic Transmitter."

centimetres were required. To obtain these small wave lengths the frequency of the sound waves ranged upwards from 20,000 cycles per second, which was well above the limits of audibility.

In previous work, accounts of which are not yet published, Dr. Langevin who initiated the subject in France, and Dr. Boyle in England, have shown that the central zones, referred to above occur in ultra-sonics and can be made available for many kinds of scientific work. The object of the present investigation is first, to determine the characteristics of the ultra-sonic beam, and to see how far Verdet's mathematical work in optics is applicable in the ultra-sonic case; secondly, to obtain absolute measurements of the ultra-sonic energy; and thirdly to investigate various ultra-sonic transmitters.

The outstanding difference between Berdet's optical case and the ultra-sonic case at present under consideration is the difference in the wave lengths involved and their relation to the "aperture" or vibrating plate. In the optical case, a wave length is extremely small in comparison with the diameter of the smallest possible aperture, whereas in ultra-sonics, the wave length λ is comparable with the diameter (d) of the oscillating plate. Due to thecomparatively large ratio (λ/d) of wave length to diameter certain assumptions made by Verdet will not hold in ultra-sonics and up to the present no precise and rigid mathematical method of investigating the problem, beyond those of Verdet and Airy has been devised.

P A R T   I.

SHAPE OF ULTRA-SONIC BEAM

1. Arrangement of Generating Circuit

All the work of this investigation was carried out with water* as the medium carrying the ultra-sonic waves. A strong, wooden T- shaped tank, fifteen feet long by five feet wide by three feet (see fig. I), was built and filled with water. The plate which was used to generate the ultra-sonic energy was excited by means of high frequency oscillating voltages, generated electrically as in "wireless" and the electrical energy was thus transformed to ultra-sonic elastic energy in the water of the same frequency as the oscillating voltages applied to the plate. To generate the required high frequency electric oscillations, use was made of three-electrode thermionic valves.

* THE AVERAGE TEMPERATURE OF THE WATER WAS 13.7°C. THE TEMPERATURE RANGE WAS FROM 11.0°C TO 16°C.

Figure 1.
WATER TANK

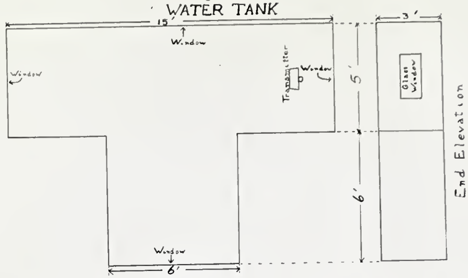


Figure 2.
ARRANGEMENT
of
GENERATING CIRCUIT

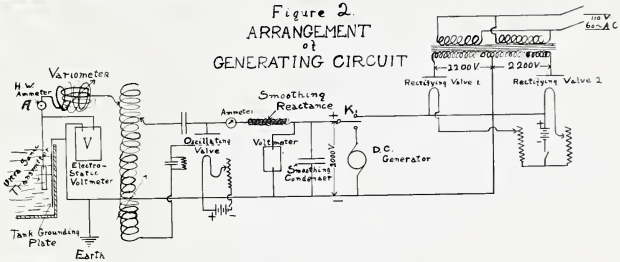


Figure 3
GENERATING CIRCUIT E.M.F.'S.

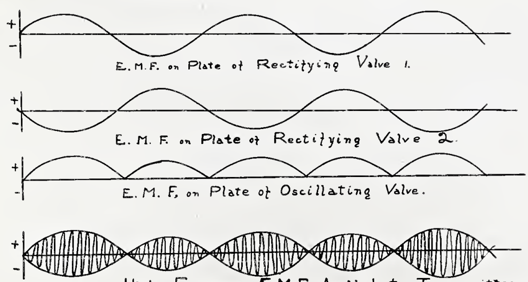


Plate #1

The transmitter served as the condenser of the oscillating circuit. The water surrounding the transmitter was connected by a grounding plate to the filament end of the oscillating inductance, and served as one plate of the condenser while the second plate was the insulated steel disc placed on the back of the transmitter and connected to the high tension side of the oscillating inductance. In this way oscillating voltages of a high frequency were impressed on the transmitting quartz plate, which was thus set into oscillation (1) and generated ultrasonic oscillations in the water. The voltages impressed on the transmitter were measured by electro-static voltmeter V, while the radio frequency current through the transmitter was measured by the ammeter A.

The uni-directional potential for the plate of the three-electrode generating valve was obtained by rectifying a 60-cycle A.C. supply. The 110-volt 60-cycle lighting circuit was transformed up to 4400 volts and rectified by two electrode valves connected in tandem as shown in figure 1. The resulting potential on the plate of the generating valve was, therefore, a series of uni-directional pulses at a frequency of 120 pulses per second, what is known as a "tonic train" (see figure 3). A partial smoothing out of these pulses of E.M.F. could be effected by the use of smoothing condenser and reactances,- as shown in figure 2. Because of this pulsating voltage on the transmitting valve the ultrasonic oscillations generated in the water were not "continuous", but were transmitted in a tonic train with a group frequency of 120 per second. The presence

(') See Section 2.

of this group frequency made it possible to detect the ultra-sonic beam with a stethescope or Broca tube by listening for the note of "note frequency" 120, and so, at times, proved very convenient. Later in the investigations, a 2000 volt D.C. generator was obtained and connected as shown in figure 2. Key K_1 allowed either the continuous or the pulsating plate source to be used.

2. Ultra-Sonic Transmitter

The transmitting plate mentioned above consisted of a quartz disc. The piezo-electric property of the quartz was used to transform the high frequency electric oscillations into longitudinal elastic vibrations. Quartz crystals were cut into plates, the cuts being made in a plane normal to the piezo-electric axis of the crystals. These quartz plates were then tested and the sections showing good piezo-electric properties were cut out and used to construct the mosaic which served as the oscillating plate in the ultra-sonic transmitter. Due to the fact that only small sections of quartz showing good piezo-electric properties could be obtained, the transmitter plates had to be built up in the form of a mosaic.

A back plate of mild steel with dimensions as shown in figure 4 (a) was turned and a large flat bottom cast iron pot was used to serve as a case. A hole six inches in diameter, the diameter of the quartz plate, was cut in the bottom of the pot. This was covered by a thin protective sheet of mica on which the quartz plate was laid. Cut (~~a~~ A) of the back plate was then fitted over the generating plate. A well insulated cable was attached to the back plate and the pot was then filled with a mixture of resin and paraffin wax. (A sketch of the transmitter is given in figure 4 (b)).

FIGURE 4
ULTRA SONIC
TRANSMITTER

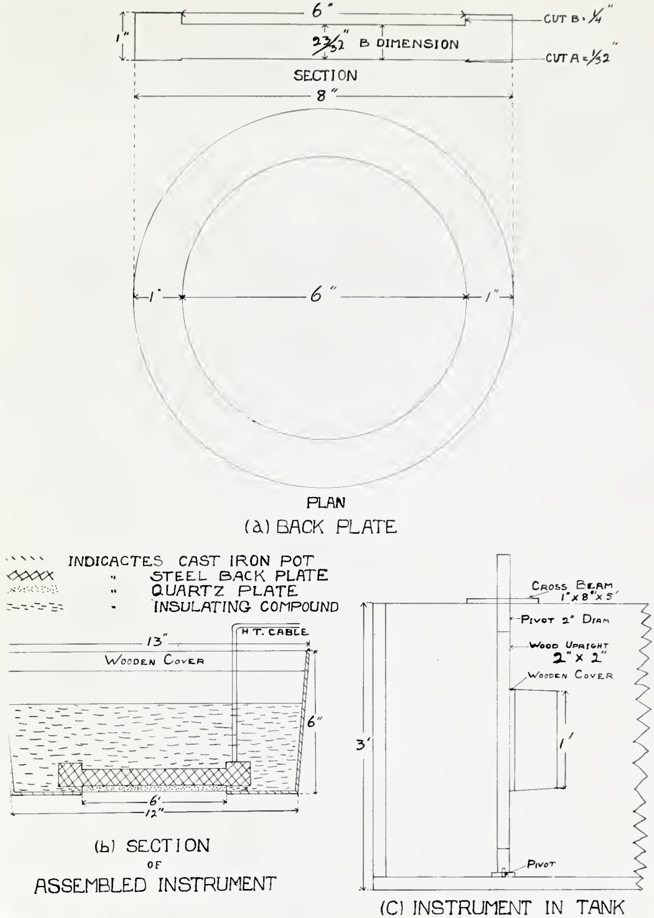


PLATE #

When the transmitter was completed it was mounted on a vertical frame, with a pivot at the bottom, at one end of the large tank and connected to the electric generating circuit, as shown in figure 2.

When an electric field is applied to such a quartz plate the piezo-electric property of the quartz produces either a small increase or decrease in the thickness of the plate, depending on the direction of the electrostatic field. Therefore, when an alternating voltage is applied to the quartz an increase in the thickness of the plate is produced during one-half cycle and a decrease during the succeeding half cycle. In this way, longitudinal oscillations are set up in the quartz plate, and these are transmitted through the mica face of the instrument and set up ultra-sonic oscillations in the water. Of course the amplitude of vibration in the quartz, and therefore also in the water, are exceedingly small.

3. Methods of Detecting Ultra-Sonic Energy

All methods of detecting the ultra-sonic energy were based on the radiation pressure exerted by sound waves. Lord Rayleigh(') has shown that a plane sound wave impinging upon a perfectly reflecting partition produces a pressure p of magnitude

$$p = 2E$$

where E = energy density. IN THE INCIDENT WAVE TRAIN

(W. Altberg('') has used this radiant pressure to measure the absolute intensity of sound radiated by a ~~Kundts~~ KUNDTS tube in air.)

There are several methods by which we have been able to detect the ultra-sonic energy, but in all these methods the radiant pressure of the ultra-sonic vibrations is utilized.

(') Phil. Mag. (6) 3 p. 338, 1902 (") Drude Annalen der Phys. 1903

X = DISPLACEMENT OF PENDULUM

(a) Cork and Light Ball Method

One of the first attempts to detect the ultra-sonic beam was to suspend inverted from the bottom of the tank a small piece of cork before the face of the ultra-sonic transmitter and at some distance from it. It was hoped that the radiant pressure on the cork would be sufficient to deflect it slightly. The power available, at first, was, however, insufficient to produce any noticeable change in the position of the cork, but later, when more power was available, a similar experiment with hollow celluloid balls was tried and proved successful. A number of celluloid "ping-pong" balls were weighted with lead shot until they would just sink when placed in the water tank. They were then suspended in a line across the tank at intervals of a few centimetres and the ultra sonic beam directed along a line perpendicular to the row of balls. The ball which was in the direct path of the beam was deflected along the beam about two centimetres. The fact that only one of the balls was deflected seemed to suggest that the ultra-sonic energy was confined to a very narrow beam. Further experiments described in section (8) below confirmed this suggestion.

The celluloid ball experiment was the most successful method of visualizing the ultra-sonic beam. The chief difficulty was to weight the balls with sufficient accuracy to make all equally sensitive to the ultra-sonic energy. In the above experiment the balls were suspended on strings about forty centimetres long and the experiment was carried out at a frequency of 130,000 cycles with 1000 volts on the transmitter. It was estimated that the radiant pressure of the beam upon the balls would be of the order of 0.4 dynes per square centimetre, and the maximum deflection obtained was ~~tow~~ two centimetres. The diameter of the ball was two centimetres so that its area normal to the path of the beam was 3.1 square centimetres. Now the weight of the ball in water must bear the same ratio to the force due to the radiant pressure on the ball as the length of the

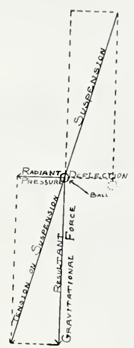
W = WEIGHT OF BALL (DYNES)
F = FORCE DUE TO RADIATION PRESSURE
l = LENGTH OF SUSPENSION
x = DISPLACEMENT OF PENDULUM

$$\frac{W}{F} = \frac{\sqrt{l^2 - x^2}}{x} = \frac{l}{x}\left(1 - \frac{1}{2}\frac{x^2}{l^2}\right) \quad \text{AND } \frac{1}{2}\frac{x^2}{l^2} = \cancel{1600}\ \frac{4}{3200} = .0012$$

WHICH IS NEGLIGIBLE IN COMPARISON WITH 1.

ball's suspension bears to the distance it is deflected. See figure 5.

Figure 5
CELLULOID BALL



Then the weight of the ball in water

$$= \frac{(3.1 \times 0.4)\ 40}{2 \times 980} \text{ gms.} = .005 \text{ gms.}$$

It is easy to see that when lead shot was used to weight the balls it was impossible to adjust them with required accuracy. Also as the volume of the balls was about 25 cubic cemtimetres, a rise in the temperature of the water of only 1°C. would be sufficient to cause the balls to float. However, the results obtained show that if sufficient care is taken in adjusting the balls, fairly satisfactory results can be obtained in finding the actual radiant pressures exerted in the beam.

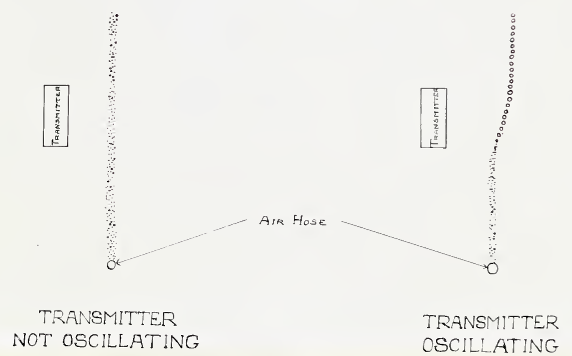
(b) Bubbles of Gas Passing Through the Beam of Ultra-Sonic Energy

A second experiment was to find the effect of the radiation pressure of the ultra-sonic beam on bubbles of gas sent through the beam. A row of very fine holes was picked in a rubber tube and the tube laid along the bottom of the tank Air was forced into the tube at a slight pressure and fine bubbles rose out of the holes. A stream of very fine small bubbles which rose very slowly was chosen and the transmitter was placed two or three inches from the stream.

At a frequency of 71,000 cycles per second with 850 volts on the transmitter there was no change in the bubbles. The frequency was then increased to 89,000 cycles and the voltage to 1000 volts. Under these conditions the bubles appeared to COALESCE and instead of a stream of very fine bubbles a single row of larger bubbles was obtained while the stream was deflected about a quarter of an inch from its original path. The experiment was repeated with the

transmitter on the opposite side of the stream and both the ~~coagulation~~ COALESCENCE and deflection were again noted. The effect produced is shown in figure 6. The experiment cannot be classed as a success, however, for these results were reproduced only two or three times in a very large number of trials. To have any success the bubbles have to rise extremely slowly and therefore be very minute. Great difficulty was experienced in getting the bubbles fine enough and in reproducing the requisite size of bubble.

FIGURE 6
BUBBLES OF AIR
EXPERIMENT



(c) Attempt to Make Visible the Path of Ultra-Sonic Beam

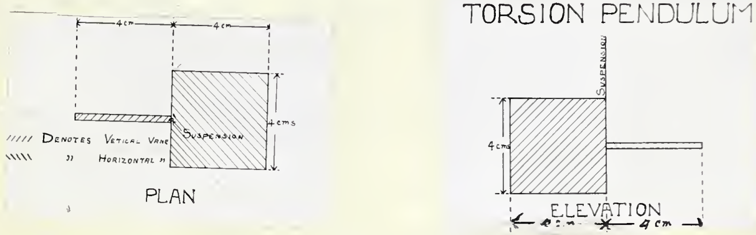
An attempt was made to render visible the path of the ultra-sonic beam. Coloring matter, potassium permanganate, was placed in the path of the beam, and it was hoped that the coloring matter, possibly, would diffuse more rapidly along the path of the beam than elsewhere. No such results were obtained when a voltage of 1000 volts, at a frequency of about 100,000 cycles per second was impressed on the transmitter. As this was the maximum voltage available at the time, the attempt was not carried further.

(d) Torsion Pendulum Method

The most successful quantitative method of detecting ultra-sonic energy was the torsion pendulum method. This method is somewhat similar to the method W. Altberg(1) used to measure the absolute intensity of sound radiated from a Kundt's tube.

A torsion pendulum was made by soldering two brass plates, four cms. square and 0.136 cms. thick, at right angles to each other, as shown in figure 7, and suspending them by a phosphor bronze strip about 30 cms. long. A similar pendulum was made from iron.

Figure 7
TORSION PENDULUM



The pendulum was suspended in the path of the ultra-sonic beam and the radiant pressure of the beam on the vertical vane deflected it. The horizontal vane merely served as a counterpoise.

This method of detecting the ultra-sonic energy was quite satisfactory. A pendulum designed to serve as an accurate instrument for the measurement of ultra-sonic energy intensities is described in section 4. below.

(e) Audible Method of Detecting Ultra-Sonic Energy

Of the above methods for detecting ultra-sonic energy, the torsion pendulum one is the most satisfactory for quantitative measurements. One other method which has proved very convenient for qualitative work is based on a listening device. A rubber nipple was placed on one end of a tube, and to the other end

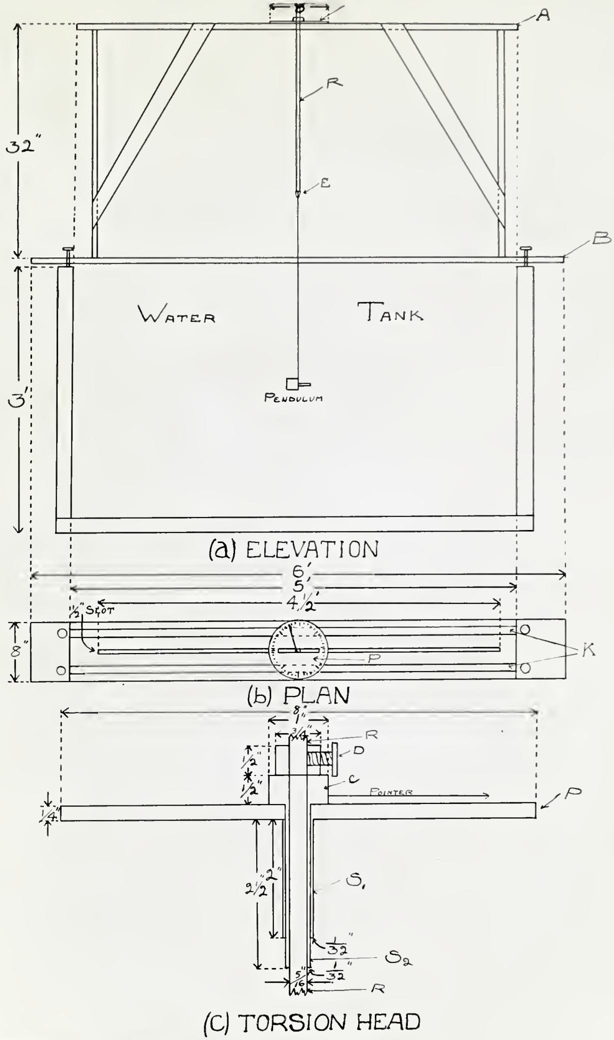
(1) Drude Annal. der Phy. 1903

of the tube was attached a stethescope. Both glass and brass tubing has been used. When the source of energy is rectified alternating E.M.F. the ultra-sonic oscillations are transmitted in a tonic train, the group frequency being twice the number of cycles per second (see fig. 3). When the nipple of the listening device is placed in the ultra-sonic beam, the radiant pressure of each group of ultra-sonic oscillations distorts it slightly and a sound impulse is produced in the tube attached to the nipple. Consequently in the present case the tonic wave train produces an audible note of 120 cycles pitch, and this note can be heard in the stethescope. By this method the course of the ultra-sonic beam was followed.

4. Description of the Torsion Pendulum used Throughout the Investigation

The preliminary work described above was carried out with a very rough Torsion pendulum. To obtain more accurate measurements a better instrument had to be devised. The pendulum was supported by a wooden framework, consisting of a shelf A attached to a baseboard B (Fig. 8(a)). The shelf was five feet long, eight inches wide and one inch thick while the dimensions of the base board were six feet by eight inches by one inch. Both shelf and baseboard had a slit four and a half feet long and one half inch wide cut down the centre. The pendulum suspension passed through this slit and on either side of ~~it~~, the upper slit, were placed two brass rails, (K,K) along which the Torsion head P could travel, (see fig. 8(b)). The plate P served as a Torsion head and was supported by the brass rails on shelf A. Through the centre of this plate (see fig. 8(c)), the sleeve S_1 was attached. The sleeve S_2, to which the collar C was attached, fitted in sleeve S_1 and was free to turn. The rod R fitted through the sleeve S_2 and could be clamped in any desired vertical position by the set screw D. The pendulum suspension F was attached to the bottom of the rod R by a small clamping chuck E. The pendulum could be placed in any required vertical position by raising or lowering the rod R, in the sleeve S_2, while horizontal adjustments were made by sliding the plate P along the rails K. To obtain readings the pendulum was first adjusted by twisting the sleeve S_2 until THE ~~its~~ vertical vane was at right angles to the path of the ultra-sonic beam. The transmitter was then set oscillating and the radiant pressure of the ultra-sonic energy on the vertical vane deflected the pendulum. The pendulum was brought back to its zero position by twisting the rod R and the sleeve S_2 in the sleeve S_1 until the torsion of the suspension brought the vertical vane of the pendulum back to the zero position. The amount of torsion in the suspension was measured by a pointer attached to the collar C, and moving over a degree

FIGURE 8



A
R
E
B
32"
3'
WATER
TANK
PENDULUM
(a) ELEVATION
6'
5'
4 1/2'
1/2" SLOT
8"
K
P
(b) PLAN
8"
1"
3/4"
R
D
1/2"
1/2"
C
POINTER
P
1/4"
2"
2 1/2"
S1
1/32"
S2
1/32"
5/16"
R
(C) TORSION HEAD

scale on the plate P. The amount of torsion in the suspension required to bring the pendulum back to its zero position is called "Pendulum ~~Deflection~~ READING" in all the following work.

In section 3 it was shown that the pendulum was deflected by a radiant ultra-sonic pressure P and, ASSUMING THAT THE PENDULUM IS A PERFECT REFLECTOR AND THAT THE ENERGY STRIKES THE PENDULUM NORMALLY.

$$P = 2E$$

where E = density of ultra-sonic energy.

This radiant pressure produces a deflecting torque T on the vertical pendulum vane and if the vane is square, as in figure 7, with sides of length "a"

$$T = Pa^2 \left\{ \frac{a}{2} \right\} = \frac{1}{2} Pa^3$$

The pendulum is restored to its original zero position by means of a restoring torque produced by the torsion in the suspension.

If θ = "Pendulum ~~Deflection~~ READING" in radians

δ = Measured ~~Deflection~~ READING in degrees

K = Torsion constant of suspension per unit length

l = Length of suspension

$$T = \frac{K}{l}\theta = \frac{K}{l}\frac{\delta}{57.3}$$

and $\therefore$ $$\frac{K\delta}{l \times 57.3} = \frac{1}{2}Pa^3 = Ea^3$$

or $$E = \frac{K}{57.3a^3 l}\delta$$

Therefore the pendulum ~~deflection~~ reading δ is proportional to the energy density E.

The pendulum vanes were made of lead. Lead was chosen because the low velocity of sound in this material allowed comparitively thin sheets to be used, for, as shown in section 9 below, a precaution must be observed ensuring a definite relation between the thickness of the pendulum vane and the wave length of the ultra-sonic energy. When all factors are considered lead is the most satisfactory material available. In section 9 it is shown that the moment

of inertia of a lead pendulum is smaller than that of a pendulum of equal reflecting power made from any other material. Consequently its period of oscillation is smaller and this is of great advantage in expediting the work.

A large number of pendulums were made with varying sizes of vanes, to suit the requirements of the different experiments. The first pendulums were made with square vanes but later circular vanes were used because the resulting symmetry greatly facilitated the investigation of the reflection and scattering of energy by the vane. Also the circular vanes were easier to make.

The suspensions used in the pendulum were phosphor bronze. Both strip and wire suspensions of varying sizes and lengths were used as suited the requirements of the different experiments. Generally the suspensions and pendulums were chosen so as to give "~~deflections~~ readings" of about 100°. The degree scale could be read to within 0.5° so that readings could easily be obtained accurately to within 1%. If the ~~deflections~~ readings were much greater than 100° considerable difficulty was experienced in handling the instrument.

5. Reflecting and Absorbing and Dissipating Screens:

All the work of this investigation was carried out in a tank fifteen feet by five feet by three feet. When it is considered that ultra-sonic oscillations have been detected some thousands of yards from the transmitter, it is evident that reflections from the ends and sides of the tank must have a considerable effect unless by some means they are avoided. An effort was made to find some material which would absorb the ultra-sonic energy and also as much as possible prevent reflections. Hair felt seemed a very promising material as it was thought that the fibrous structure would dissipate much of the energy passing through the felt, and whatever energy was reflected would be scattered.

Two thicknesses of felt were tried: half an inch thick and quarter of an inch thick. The felt was tacked to wooden frames which were placed at various positions in the tank. U[illegible] detector it was found

that no energy passed through either thickness of felt, but a considerable portion of the incident energy was reflected. The absorbing quality of these felt mats has been made use of to screen off ultra-sonic energy from certain localities in the tank. The quarter inch felt was just as effective a screen as the heavier half inch felt.

In all experiments a felt mat was used to prevent any possible energy reflected from the end or sides of the tank, behind the transmitter, from affecting the pendulum. This mat was stretched across the tank at the face of the transmitter, the transmitter projecting through a hole in the centre. When this mat was in position the ~~deflection~~ reading of the pendulum was decreased from 36° to 31° for one trial and on another occasion from 34° to 31°, so apparently it was necessary and also quite effective. The mat was left in position permanently.

Experiments were also tried with mats placed along the sides and bottom of the tank and the surface of the water but no change was noticed in the ~~deflection~~ reading of the pendulum. The later investigation of the shape of the ultra-sonic beam showed that the beam was so narrow that no energy could reach the sides of the tank until it had passed very considerably beyond the pendulum. In fact most of the ultra-sonic beams investigated had spread over only one half the cross sectional area of the tank by the time they reached the far end of the tank. Therefore, except for certain particular cases, it was not necessary to have absorbing mats covering the sides of the tank and the surface and bottom of the water.

Later work showed, however, that reflections from the far end of the tank distorted the distribution of energy in the incident beam very materially, and also decreased its intensity. An effort was therefore made to eliminate this reflected energy. A number of felt mats were fastened parallel to each other in a light wooden framework and placed at the end of the tank furthest from the transmitter, at an angle to the central axis of the tank, as shown in figure 9.

It was hoped that the ultra-sonic beam would follow the path of the dotted line in figure 9 and thus gradually dissipate itself, instead of being reflected back on its path.

In order to determine the effect of these dissipating screens the distribution of the ultra-sonic energy in the tank was investigated with the torsion pendulum. The position of maximum energy intensity in a vertical plane was first obtained by raising and lowering the pendulum until the position in which it gave a maximum reading ~~deflection~~ was located. The pendulum was then moved in a horizontal plane across the tank and by observing its deflection in various positions a horizontal cross section of the energy distribution in the ultra-sonic beam was obtained. Two such sections were taken, one when the end of tank alone was reflecting the incident energy and the other when the dissipating screens were in position. Any discrepancies in the two sets of readings must be due to the effect of the dissipating screens. This method of investigating the distribution of ultra-sonic energy is described, in greater detail, in section 7 (d) below.

The results obtained are tabulated in Table I and plotted in figure 10.

FIGURE 9
POSITION OF
DISSIPATING SCREENS

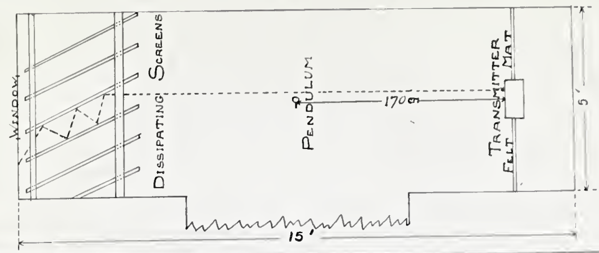


FIGURE 10
SECTION SHOWING EFFECT OF
DISSIPATING SCREENS

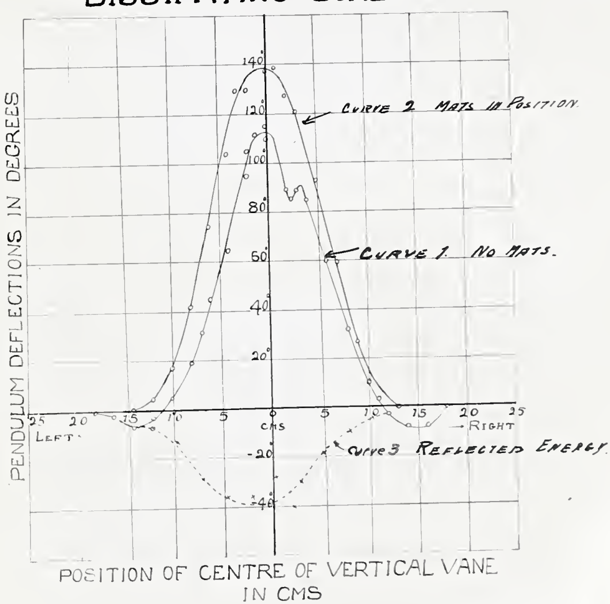


PLATE 5

In Table I the distance in centimetres of the centre of the vertical vane of the pendulum from the central axis of the ultra-sonic beam is quoted under the column headed "Position of the centre of the vertical vane". The corresponding pendulum ~~deflections~~ readings are quoted in the column headed "Pendulum ~~Deflections~~ READINGS". In figure 10 the abscissae represent the position of the centre of the vertical vane of the pendulum while the ordinates represent the pendulum ~~deflections~~ readings. Curve 3 of figure 10 represents the reflected beam when the mats are not present. It was obtained by taking the difference in the ordinates of the two sections. The sections were taken at a distance of 170 centimetres from the transmitter, approximately midway between the transmitter and the dissipating screens.

TABLE I.

Series I | No Mats Present | Centre of tank at 0 cms.

Position of centre of vertical vane.	Pendulum ~~Deflection~~ READINGS.
2.5 cms. left	105 degrees
0.5 " "	115 "
1.5 " "	112 "
1.5 " right	89 "
3.5 " "	85 "
2.5 " "	89 "
5.5 " "	60 "
7.5 " "	32 "
9.5 " "	10 "
11.5 " "	-3 "
13.5 " "	-8 "
15.5 " "	-8 "
17.5 " "	0 "
0.5 " left	110 "
2.5 " "	95 "
4.5 " "	71 "
6.5 " "	45 "
8.5 " "	19 "
10.5 " "	5 "
12.5 " "	-3 "
14.5 " "	-7 "
16.5 " "	-2 "
18.5 " "	0 "

TABLE I

Series I[1]

Mats in Position

Position of centre of vertical vane	Pendulum Readings
3.5 cms. Left	130 degrees
1.5 " "	843 "
0.5 " Right	140 "
2.5 " "	121 "
4.5 " "	93 "
6.5 " "	60 "
8.5 " "	27 "
10.5 " "	3 "
12.5 " "	0
1.5 " "	128 "
6.5 " Left	138 "
2.5 " "	130 "
4.5 " "	104 "
6.5 " "	75 "
8.5 " "	42 "
10.5 " "	17 "
12.5 " "	4 "
14.5 " "	0

From figure 10 it appears that the reflected energy amounts to at least twenty-five percent of the incident energy at 170 centimetres from the transmitter. The screens increase the pendulum deflection by about thirty per-cent.

The magnitude of the energy reflected by the far end of the tank was not discovered until the work described in section 7 below wasalmost completed. In the earlier work, therefore, corrections have been made, where necessary, to allow for this reflected energy. These corrections are based on the results shown in figure 10. In the later work, that described in Part II et seq., the dissipating screens were in position at all times.

6. Resonant Frequency of the Transmitter:

It was found in the preliminary work that with any transmitter the frequency of the ultra-sonic energy had a very marked effect on the pendulum ~~deflections~~ readings. The pendulum was placed, for any given distance from the transmitter, at the point of maximum energy intensity in the ultra-sonic beam, i.e. on the axis, and the frequency of the ultra-sonic was varied by varying the inductance of the electrical oscillating circuit (see fig. 1). The point of maximum intensity was located by moving the pendulum horizontally across the tank, and also up and down in a vertical direction, until the position in which the pendulum showed a maximum deflection was found.

With a transmitter as described in section 2, an experiment was carried out with 1000 volts across the transmitter and a range of frequencies from 100,000 cycles per second to 185,000 cycles per second. The frequencies were measured by a Hertzian wave meter' calibrated to give readings correct to within one percent. The results obtained are tabulated in Table II and plotted in figure 11.

TABLE II.

Resonant Point of Transmitter

Voltage on Transmitter = 1000 volts.

Frequency	Pendulum ~~Deflection~~ Readings.
185,000 cycles per second	2.5 degrees
175,000 "	5.0 "
167,500 "	25.0 "
160,000 "	82.0 "
152,500 } " 154,500 } "	92.0 " Two frequencies detected
150,000 "	154.0 "
148,000 "	143.0 "
147,000 "	161.0 "
143,000 "	189.0 "

FIGURE 11

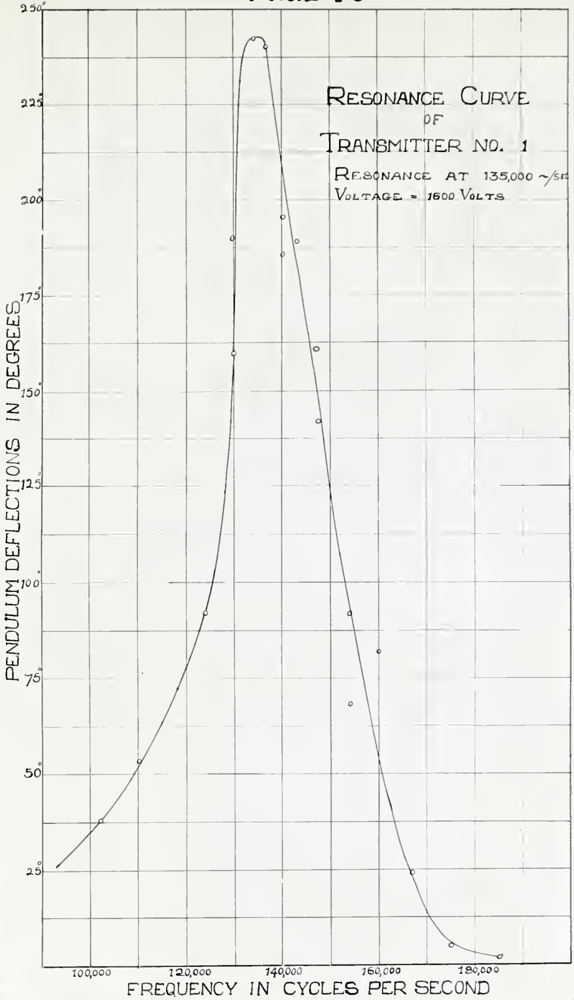


PLATE 6

TABLE II. (Contin.)

Frequency	Pendulum ~~Deflection~~ Readings.	
140,000 cycles per second.	195.0 degrees	
140,000 "	186.0 "	
137,000 "	230.0 "	
137,500) "		
136,500) "	239.00 "	Two frequencies detected.
134,000 "	244.0 "	
130,000 "	160.0 "	
124,000 "	92.0 "	
118,500 "	88.0 "	
115,000 "	73.0 "	
102,000 "		
104,000 "	38.0 "	Two frequencies detected.

It will be noticed in figure 11 that many points were a considerable distance off the main curve, and certain identical frequency settings did not give quite the same ~~deflections~~ readings. This lack of repetition was probably due to inaccuracies in measuring the frequency. The Hertzian wave meter used was accurate only to within about one percent of the correct values. With the frequencies used in this experiment a variation of one percent would amount to about 1000 cycles. Few of the points in figure 11 are more than 1000 cycles from the main curve.

The experiment shows that this particular instrument passed through a condition of maximum energy emmission at a frequency of 135,000 cycles per second. As the frequency of the electric oscillations applied to the transmitter increased towards this value, the ultra-sonic energy radiated by the instrument increased rapidly to a maximum value at a frequency of 135,000 cycles per second, and with further increases in frequency the energy radiated rapidly decreased. This indicates that at a frequency of 135,000 cycles per second, the frequency of the electric oscillations applied to the instrument coincided with the natural frequency for free longitudinal vibrations

in the transmitter, and a resonance condition occurred. The region of the resonant frequency,-the "peak",-is much sharper than was expected. In fact it is difficult to keep the frequency constant to the resonant value for any transmitter oscillating circuit. This will be referred to later in Part III.

7. Shape of the Ultra-Sonic Beam:

(a) Verdet's Work for Optical Case:

It was mentioned in the introduction to this paper that the case of ultra-sonic oscillations generated by an oscillating disc is analagous, mathematically, to the optical phenomenon of a parallel beam of light passing through a small circular opening. E. Verdet' has treated the optical case mathematically and because of its bearing on the present investigation, his results are here quoted.

Verdet assumed that parallel rays of light pass through a circular opening in a direction normal to the plane of the opening. As the phenomenon is symmetrical the effect will be the same in all planes which are normal to the opening and include a diameter of it. Verdet considers the effect in one of these planes. The method followed was that of Knochenhauer".

Figure 12

'.E. Verdet: Lecons d'Optique Physique,-Tome 1 page 301.
" Knochenhauer: Die Undulations Theorie des Lichtes, Berlin 1839, p 22

Verdet developed an expression for the intensity of illumination at a point M (fig. 12) very distant from an opening AOB,

When

θ = Angle between the line joining M to the opening and the central axis.

λ = Wave length of energy under consideration.

R = Radius of opening

$$m = \frac{\pi R \sin\theta}{\lambda}$$

I^2 = Intensity of illumination at M

Then

$$I^2 = \pi^2 R^4 \left[1 - \frac{m^2}{2} + \frac{m^4}{(2.\cancel{3})^2.3} - \frac{m^6}{(2.3.\cancel{4})^2.4} \cdots\cdots\right]^2$$

As m increases, that is as θ increases, the value of the bracketed expression becomes alternately positive and negative and therefore must pass through zero values. Therefore as θ increases the intensity of illumination varies from a maximum when $\theta = 0$, down to zero intensity, then increases to a maximum and again falls off to zero, etc. We see, therefore, that the energy is distributed from the opening in zones of maximum intensity separated from one another by zones of zero intensity. The values of m which give maximum and minimum intensities have been calculated by Verdet and his results are quoted in Table III together with the relative values of various maximum intensities.

TABLE III

	~~θ~~ $\frac{m}{\pi}$	Intensity
First Max.	0	1.0
First Min.	0.610	0
Second Max.	0.819	0.0175
Second Min.	1.116	0
Third Max.	1.333	0.00415
Third Min.	1.619	0
Fourth Max.	1.847	0.00165
Fourth Min.	2.120	0

It is to be noted that the intensities in the second and subsequent maxima are negligible in comparison with the intensity of the first maximum. That is, most of the energy is confined to a central zone whose angular width may be determined from the value of "m" at the first minimum. Taking the value of "m" for the first minima from Table III and substituting, we get:

$$\sin\theta = .610\frac{\lambda}{R} \quad \text{or} \quad 1.22\frac{\lambda}{D}$$

Where D is the diameter of the opening.

Airy' has also treated the matter mathematically, although not quite so rigorously as has Verdet. The value Airy gives for θ at the first minimum is

$$\sin\theta = \frac{\lambda}{D}$$

This differs from Verdet's results by the constant 1.22. Airy points out that the criterion for obtaining the central zone of energy is that λ must be small in compariosn with D, and states that could this condition be established for sonic waves, the same interference phenomena should occur.

When ultra-sonic oscillations are used this condition is readily attainable For example, using the transmitter described in section 2, at its resonance frequency of 135,000 cycles per second as obtained in section 6, we have,

$$R = 7.65 \text{ cms.}$$

$$\lambda = \frac{1.5 \times 10^5}{1.35 \times 10^5} = 1.11 \text{ cms.}$$

(1.5×10^5 = velocity of sound in water in cms. per second)

therefore $\sin\theta = .610 \times \frac{1.11}{7.65} = .088$ RADIANS

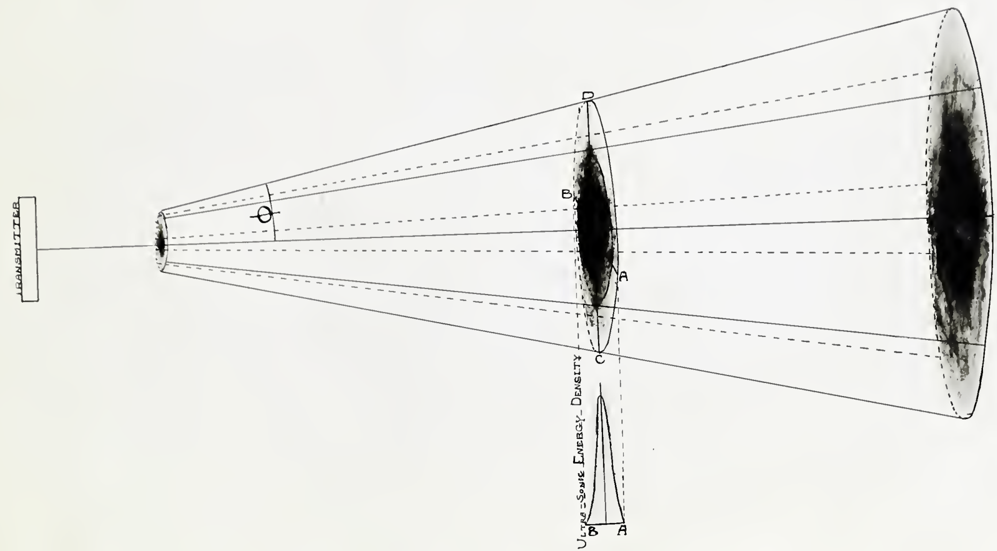
or $\theta = 5^\circ$ approximately.

A sketch of the central zone is given in figure 13. This zone has the form of a central cone with maximum energy intensity along its central axis, and moving away from this axis the energy gradually diminishes to zero. The energy intensity is represented in figure 13 by shading ~~the thickness of the contour lines, and by shading~~.

' Loc. Cit.

Figure 13

VERDET'S CENTRAL ZONE



The phenomenon may be investigated experimentally by using a torsion pendulum to determine the energy distribution in the planes AB and CD. A replica of a section obtained with a torsion pendulum is included in the figure.

(b) <u>Certain Factors which Verdet Omits, but which Should be Considered in the Ultra-Sonic Case.</u>

(1) It is to be noted that Verdet assumed that the point M (fig. 12) was very distant from the opening. Let us suppose that ten centimetres would be a sufficient distance to allow, and find what distance would correspond to this in the ultra-sonic case. Assuming that the light wave passing through the aperture had a wave-length of 5×10^{-5} centimetres, a distance of ten centimetres would equal 2×10^{5} wave-lengths. Much of the work in this investigation was done with an ultra-sonic wave-length of 1.11 cm., and to obtain a distance comparable with ten centimetres for the optical case, the ultra-sonic work would have to be done at as great a distance as 2.22 kilometres from the ultra-sonic transmitter. This, of course, would be quite impossible in any laboratory experiments, but even at distances as small as two and three metres from the transmitter, results for the angle of the ultra-sonic beam have been obtained which coincide fairly well with Verdet's expression.

(2) Another factor which should be taken into consideration in the ultra-sonic case is the question of obliquity.

FIGURE 14

To get the combined effect at O (fig. 14) of energy radiated from all points of the transmitter between A and B we must obtain the components along OS of these vibrations. In other words the obliquity of the path AO and BO must be considered. The effect of obliquity has been omitted by Verdet, because in the optical case the distance from the transmitter, or aperture, must be many wave lengths, and therefore the obliquity factor would NOT be appreciable. This may also be the case for ultra-sonic oscillations at positions very distant from the transmitter but in the present investigation the distances under consideration are only a moderate number of wave-lengths and it is probable that the effect of the obliquity is significant.

(3) A third factor which arises in the sonic problem is the damping effect the vibrations will experience due to the viscosity of the medium through which the vibrations are transmitted. If sound vibrations are emitted as plane waves from a source whose motion is expressed by the relation

$$A = a_0 \cos 2\pi n t$$

then at a distance x from the source the amplitude of the sound vibrations will be diminished by the viscosity damping. In his "dynamical Theory of Sound" (page 186) Lamb has developed the following expression for plane sound vibrations at a distance x from their source

$$A_x = a_0 e^{-\frac{8\pi^2 n^2 y}{3dV^3} x} \cos 2\pi n\left(t - \frac{x}{V}\right)$$

Where x is distance from source

a_0 " initial amplitude.

y " coefficient of viscosity of medium

d " density ~~of viscosity~~ of medium

n " frequency of sound vibrations.

V " velocity of sound in ~~water~~. the medium

For water $d = 1.00$

$y = .017$ C.G.S. units (approximately)

(for water) $v = 1.5 \times 10^5$ cms per second.

And the resonant frequency of the transmitter described in section 2 above

$$n = 1.35 \times 10^5 \text{ cycles per second}$$

Under these conditions, therefore, we get

$$A_x = a_0 e^{-2.4 \times 10^{-6} x} \cos 2\pi n \left(t - \frac{x}{v}\right)$$

Assuming that Lamb's equations are applicable at ultra-sonic frequencies and also that the above values of d, y, and v, may be used when dealing with very high frequency sound waves, we see that to diminish the amplitude of vibration by only 1% of its initial value the distance from the transmitter must be of the order of forty or fifty metres. At distances of two or three metres such as might be obtained in the tank in which the investigations considered in this paper were carried on, the effect of viscosity would be very slight. When, however, the propagation of an ultra-sonic beam over distances of a few kilometres is being considered the effect of viscosity is of primary importance for this effect increases exponentially with the distance from the source of the sound vibrations.

Whether or not Lamb's equations apply to ultra-sonic vibrations is a matter for experimental investigation. As yet no data on this point has been obtained.

(c) Formation of Beam

The fact as mentioned in sub-section (b 2) above, that Verdet's mathematics is applicable only at great distances from the transmitter becomes very evident when we consider the diffraction effects which occur in the immediate neighborhood of the transmitting plate. An indication of the phenomena which occur within a distance of a few centimetres of the transmitter may be obtained from a consideration of elementary interference theory.

FIGURE 15

Let AOB (fig. 15) represent the diameter of a transmitting plate and let ON be the normal through the centre of this plate (i.e. the axis of the ultra-sonic beam). Then there will be some point P_1 on this axis at which the following relation will hold

$$AP_1 - OP_1 = \frac{\lambda}{2}$$

where λ = ultra-sonic wave length.

At the position P_1 as determined from the above equation, the transmitting plate will subtend one-half wave zone and therefore the energy intensity will be a maximum. Between this point P_1 and the transmitter there will be a point P_2 at which the transmitter subtends two half wave zones. That is

$$AP_2 - OP_2 = 2\left(\frac{\lambda}{2}\right)$$

and at this point there will be a minimum energy intensity.

Similarly between P_2 and the transmitter there will be points P_3, P_4 etc. at which three, four, or more half wave zones are subtended. At points where an odd number of half wave zones are subtended there will be maximum energy intensity while minimum intensity will result when an even number of half wave zones is subtended. We may now write the general equation for the above points P as

$$AP - OP = n\frac{\lambda}{2} \qquad (1)$$

When n is odd a position of maximum energy intensity is obtained and when n is even a minimum intensity occurs.

The following symbols will now be used:

λ = ultra-sonic wave length.

R = radius of transmitting plate.

OP = d = distance from transmitter.

Substituting these symbols in equation (1) and solving for AP, we get

$$AP = d + n\frac{\lambda}{2}$$

We also have the relation:

$$AP^2 = R^2 + d^2$$

$$\text{or} \quad \frac{n^2\lambda^2}{4} + n\lambda d = R^2$$

$$\therefore \quad d = \frac{R^2}{n\lambda} - \frac{n\lambda}{4}$$

The above diffraction phenomenon cannot exist beyond the point at which the transmitter subtends one half wave zone. Therefore the largest value of d which need be considered is that at which n = 1. For the transmitter described in section 2, operating at its resonant frequency of 135,000 cycles as determined in section 6, we have the following values of R and λ

$$R = 7.65 \text{ cm.}$$

$$\lambda = \frac{1.5 \times 10^5}{1.35 \times 10^5} = 1.11 \text{ cm.}$$

(1.5×10^5 cms per second is velocity of sound in water)

$$\text{therefore } d = \frac{(7.65)^2}{1.11} - \frac{1.11}{4} = 48 \text{ cm (approx)}$$

In this particular case, therefore, at distances from the transmitter greater than about 50 cms. a beam formation similar to that considered by Verdet would, presumably, occur, but within 50 cms. of the transmitter an al-

together different phenomenon is met with. As we approach the transmitter along the central axis of figure 15, we pass through points of maximum and minimum energy intensity.

The above is merely an approximate treatment of the energy distribution along the central axis, for the effects of obliquity and viscous damping have not been considered. The factors which would have to be considered in developing a general mathematical expression for the energy intensity at any point in the immediate vicinity of the transmitter are complex and very serious mathematical difficulties are met with. As yet no satisfactory, rigid, mathematical solution of the problem has been developed. Experimental data has been obtained, however, and evidence of some interesting diffraction figures has been found.

The instrument used to investigate the distribution of ultra-sonic energy near the face of the transmitter was the listening device described in section 3 (c). The listening tube was fixed in a stand in such a way that the nipple could be placed in any required position in the tank and its longitudinal, horizontal and vertical position with respect to the transmitter, noted. This stand was very similar to the torsion pendulum stand described in section 4.

To investigate the distribution of ultra-sonic energy the nipple of the listening device was first placed on the central axis at a distance of about 1 centimetre from the face of the transmitter. The nipple was then moved to the right and left of this central axis always being kept in the same horizontal plane. Variations in the intensity of the ultra-sonic energy were detected by the corresponding variations in the intensity of the sound heard in the stethescope. In this way positions of maximum or minimum energy intensity could be located and a horizontal cross-section of the energy distribution obtained. Similar sections of the distribution of energy were taken at increased distances from the transmitter. Sections were taken every centimetre until a distance of forty centimetres from the transmitter was attained. At this

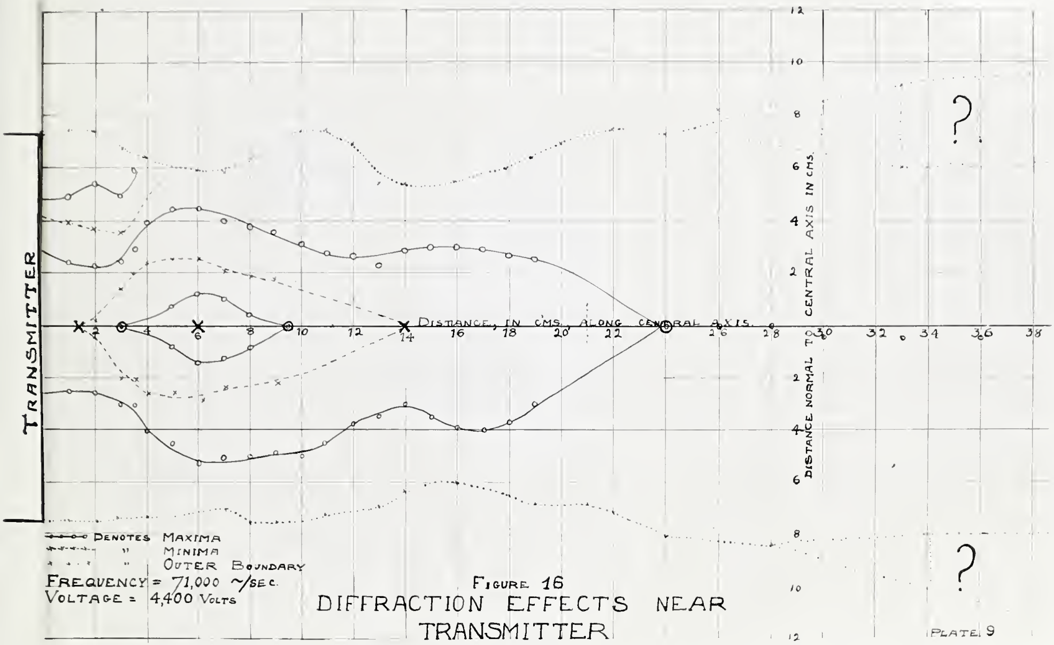


FIGURE 16
DIFFRACTION EFFECTS NEAR TRANSMITTER

PLATE 9

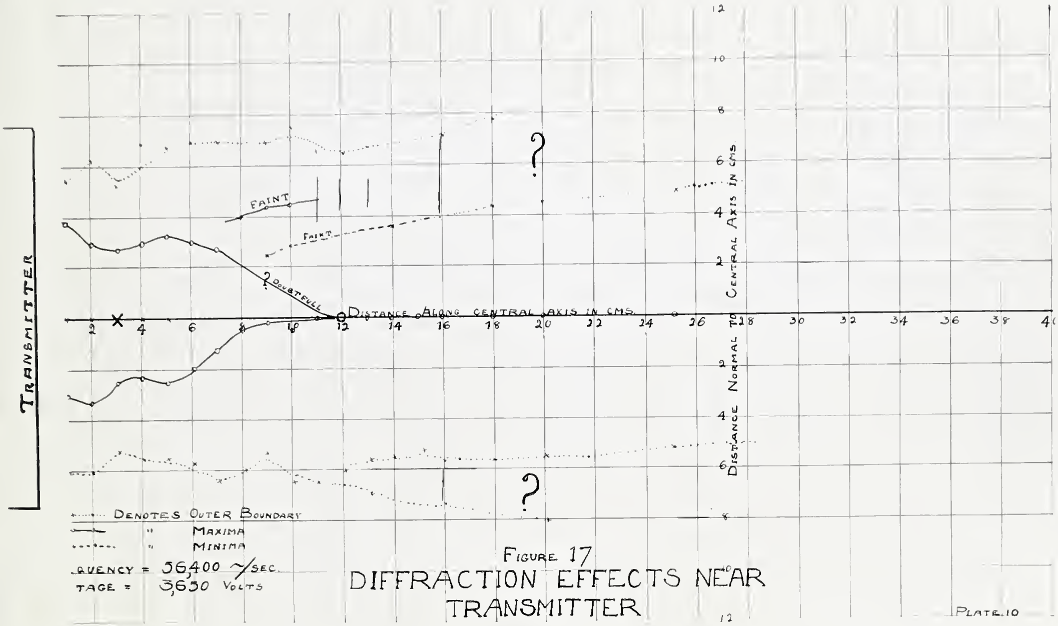


FIGURE 17
DIFFRACTION EFFECTS NEAR TRANSMITTER

distance the central beam of energy appeared to be definitely formed.

The distribution of energy was investigated at two different frequencies, namely, 71,000 cycles per second and 56,400 cycles per second. The results obtained are plotted in figures 16 and 17 respectively. In these figures the points of maximum intensity are marked by circles and those of minimum intensity by crosses. Corresponding maxima are joined by continuous lines and minima by a broken line. It was found that when the nipple was moved in and out along the central axis a point of maximum intensity was obtained where-ever two lines of maxima met, and also, minimum intensity was obtained when two lines of minima met. There is a slight lack of symmetry in both figures 16 and 17 which, no doubt, is due to imperfections in the transmitter.

From these figures it appears that in the immediate vicinity of the trans-mitter there are symmetrical zones of maximum and minimum intensity extending to a distance of a few wave lengths from the transmitter. It is to be noted that these energy zones become more complex and the distance to which they extend increases as the frequency of vibration of the ultra-sonic energy in-creases. It is beyond these zones that diffraction effects approximating to those of Verdet are obtained. The central beam commences at a distance of about thirty centimetres from the transmitter when the frequency of vibration is 71,000 cycles per second, and at a distance of about sixteen centimetres when the frequency of vibration is 56,400 cycles per second. There is also a slight indication that at these distances the outer zones discussed by Verdet are forming. The dotted lines of figures 16 and 17 represent the apparent boundary of the energy field. This boundary became rather indefinite after the region of complex zones was passed and there were slight indications that, in addition to the central beam which could be distinctly detected, there was also present an outer zone in which the energy intensity was small. The nipple detecting

~~region of complex zones was passed and there were slight indications that, in addition to the central beam which could be distinctly detected there was also present an outer zone in which the energy intensity was small.~~ ~~The nipple detecting device~~ device. was not sufficiently sensitive to definitely establish the presence of any outer zones. The presence of these zones, however, has been established by means of the torsion pendulum and they will be discussed in sub-section (e) below.

Addendum, - ~~The~~ In the above the transmitter consisted of a quartz mosaic. Undoubtedly different portions of the mosaic had slightly different piezo-electric properties and therefore the amplitude of vibration at the face of the instrument would vary slightly. It is virtually impossible to obtain a piece of quartz of large area and uniform piezo electric properties and as a result certain discrepancies between theoretical and experimental results are bound to occur.

(d) Experimental Method, OF INVESTIGATING the Central Beam

(1) Preliminary trials:

In order to determine the shape of the ultra-sonic beam the torsion pendulum, described in section 4 was used. The stand was placed across the tank at the required distance from the transmitter and the pendulum was placed as nearly as possible on the axis of the ultra-sonic beam. The pendulum was moved horizontally across the tank by sliding the plate P (fig. 8) along the shelf A until a maximum ~~deflection~~ reading was obtained. The vertical maximum was then located by adjusting the rod R in the sleeve S_2 (fig. 8) until the position which produced maximum ~~deflections~~ readings of the pendulum was found. As soon as the point of maximum ultra-sonic intensity was located, in this way, both vertical and horizontal sections of the beam could be obtained.

The first section was taken at a distance of seventy-three centimetres from the transmitter with a lead pendulum, such as has been already described, the vanes of which were three centimetres square. The voltage impressed on

Figure 18
PRELIMINARY BEAM SECTION

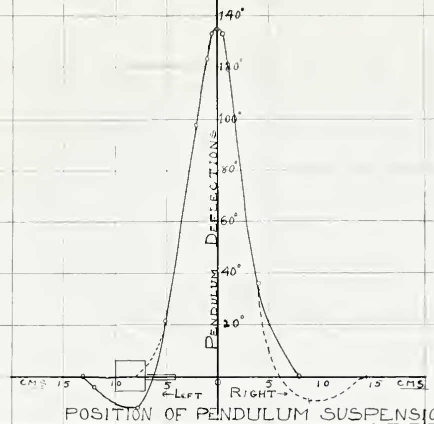


Figure 19
FINAL HORIZONTAL SECTION
(SEE TABLE IV)

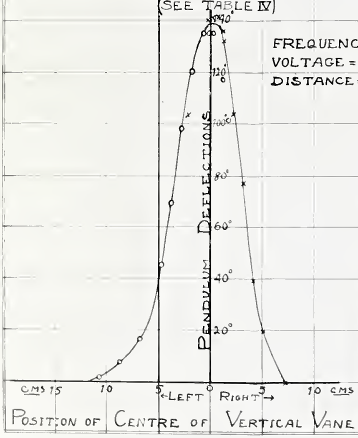
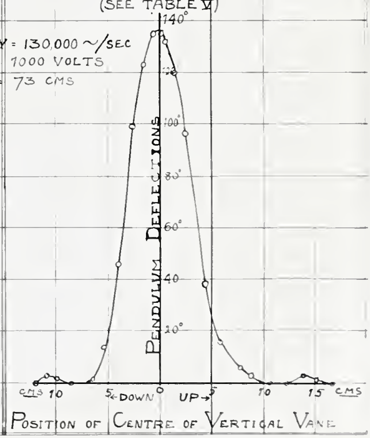


Figure 20
FINAL VERTICAL SECTION
(SEE TABLE V)



the transmitter was kept constant at 1000 volts and the frequency kept constant at 135,000 cycles per second. The pendulum was moved in a horizontal plane, from the position of zero reading ~~deflection~~ through the point of maximum intensity, to the position of zero reading ~~deflection~~ on the other side, readings being taken every two or three centimetres. The results obtained are ~~are~~ plotted in figure 18, the abscissae representing the horizontal position in centimetres of the pendulum suspension, while the ordinates represent the pendulum readings ~~deflections~~. The position of the suspension was measured by a centimetre scale on the base B (fig. 8) of the torsion pendulum stand. In figure 18, scale reading 0 represents the centre of the tank.

Precaution (a)

Certain experimental errors are shown in figure 18, the most noticeable one being the discrepancies caused by the horizontal counterpoise vane of the pendulum. The experiment was first carried out with the vertical vane of the pendulum to the observer's left looking along the path of the ultra-sonic beam away from the transmitter. The readings obtained are represented by the full line of figure 18. It will be noticed that at six centimetres to the left of the centre of the tank (scale reading 6 L) a decided negative reading ~~deflection~~ was obtained. When the pendulum suspension was in this position the vertical vane was to its left and entirely out of the energy field, while the horizontal vane was to the right and still in the ultra-sonic beam. The actual position of the pendulum is sketched in figure 18.

The apparent negative deflections of the vertical vane were, no doubt, caused by the ultra-sonic energy of the main beam striking the horizontal counterpoise vane and giving it a positive deflection. To check this, the pendulum was reversed so that the vertical vane was to the right of the suspension and in the energy field while the horizontal vane was to the left and

out of the beam. The readings obtained under these conditions are shown by the dotted line in figure 18. With the pendulum in its reversed position the readings to the right of the beam were also repeated and the results are shown by the dotted line at the abscissae readings fifteen to five, right. It will be noticed that when the pendulum was in its reversed position, the negative readings were transferred from the left to the right side of the beam thus showing definitely that they were due to the ultra-sonic energy striking the horizontal counterpoise vane, while the vertical vane which should have received this energy was out of the energy field.

From the results quoted above, it is evident that the horizontal vane must be kept in as weak an energy field as possible. This was accomplished by keeping the vertical vane of the pendulum pointed towards the centre of the ultra-sonic beam. To do this it was necessary to reverse the pendulum whenever the maximum point of the beam was passed, which necessitated a new method of designating the pendulum positions.

In the survey of the beam section referred to above, the pendulum positions were designated by the position along a horizontal scale of the pendulum suspension. In the section given in figure 18 it was assumed that the readings ~~deflections~~ were proportional to the ultra-sonic energy along SS_1. This, however, was not exactly the case as the deflections were due to the integral of the ultra-sonic energy over the face of the vertical vane, less a correction for the energy striking the horizontal counterpoise. To accomplish this integration for each reading in the tables given below would be a very laborious task. As the immediate object was not to obtain absolute energy intensities but rather to get an idea of the relative distribution of energy in the beam it was found sufficient to consider that the pendulum deflections were due to a pressure at the centre of the vertical vane, which was the average pressure over the face of the vane. In the following beam sections

the necessary corrections were applied to the position of the pendulum suspension in order to quote exactly the position of the centre of the vertical vane for plotting the curves. The actual integration, mentioned above is carried out in a later section dealing with "The Absolute Intensity of Ultra-Sonic Energy", and the precautions just described were carefully attended to.

Precaution (b)

Another point to be taken into account when obtaining the survey of a beam section is the narrow peak of maximum intensity that occurs. Figure 18 shows that in that particular section the region of maximum intensity was only about one centimetre wide. To obtain satisfactory beam sections it was therefore necessary, for the sake of accuracy, to use a pendulum with vane dimensions of about a centimetre. A pendulum with <u>circular</u> (instead of square) vanes, one centimetre in diameter and 0.254 centimetres thick was made. Fine, short pointers were attached to assist in locating the zero position of the pendulum. The outstanding disadvantage of this pendulum was the relatively large correction required to allow for the energy striking the horizontal vane. The area of the vertical vane of the pendulum was about 0.8 square centimetres while the effective area of the edge of the horizontal counterpoise was 0.254 square centimetres. This meant a correction for the horizontal vane amounting to about thirty percent of the effect due to the vertical vane, or about forty-three percent of the resultant ~~deflection~~ reading. However, as this was a constant factor which could be applied to each reading it did not affect the <u>shape</u> of the section, and as mentioned before, for our present purposes, only the relative distribution of energy was required.

This one centimetre pendulum proved to be a very convenient one to work with as it had a quick period of oscillation and the water resistance was sufficient to allow it to oscillate in a critically damped condition. Of course

decreasing the area of the vertical vane decreased the ~~deflections~~ readings obtained, but this loss in sensitiveness was overcome by using finer suspensions. With a suspension of 0.0025 inch phosphor bronze strip the pendulum was quite satisfactory for work at the resonant frequency of the transmitter, although at lower frequencies, pendulums with vanes two or three centimetres in diameter had to be used.

Precaution (c)

In addition to the above precautions the effect of the thickness of the pendulum vane must be considered. Theoretical considerations show that the thickness of the pendulum vane, as compared with the wave length of the ultrasonic energy, has a decided effect upon the proportion of energy reflected by the pendulum vane and therfore upon the radiant pressure to which the pendulum is subjected. This matter is considered in detail in section 9 below. It was found that the reflection from the pendulum is almost perfect, provided that the thickness of the vertical vane lies between 0.1 and 0.4 wave lengths. The thickness of the pendulum vanes used in all the following work lies between these limits.

II Final Trials.

Taking due regard to all precautions noted above, the section given in figure 18 was repeated with the one centimetre ~~one~~ circular vane pendulum. The readings are tabulated in table IV and plotted in figure 19. In table IV the positions of the suspension are quoted, and to obtain the position of the centre of the vertical vane a correction equal to the radius of the vane is added to or subtracted from the suspension positions as the case may require. In figure 19, the positions of the centre of the vertical pendulum vane are represented as abscissae while the ordinates represent the pendulum ~~vane deflections~~ readings. The readings obtained with the vertical vane pointing to the right

are marked with circles, while those obtained with the vertical vane pointing to the left are marked with crosses. The discrepancies between the "circle" and "Cross" readings show the necessity of keeping the horizontal vane in a weak energy field. To the right of the peak of the section the horizontal vane is the stronger field, for the circle readings, and so these readings are smaller than those of the other set. To the left of the peak the opposite conditions hold and the "circle" readings are the larger. The slight discrepancies in the peak readings are quite possibly due to slight fluctuations in the frequency of the electrical generating circuit. Figure 9 of section 6 shows that the frequency at which the above section was taken, viz. 130,000 cycles per second, is on a very steep part of the energy curve, and slight fluctuations in frequency would produce considerable changes in the intensity of the ultra-sonic energy.

TABLE IV

ultra-sonic beam section--- 73 cms. from transmitter.

voltage on transmitter --- 1000 volts.

frequency --- 130,000 cycles/sec.

pendulum, lead with circular vanes one centimetre in diameter.

suspension --- 0.002" phosphor bronze strip 80 cm. long.

Position of Suspension	Correction	Position of Centre of Vertical vane	Pendulum ~~Deflections~~ Readings
Vertical	Vane to Right of	Suspension.	
1.0 cms. Right	-0.5 cm	1.5 cms. Right	118°
1.5 " "	do.	2.0 " "	102°
0 " "	"	0.5 " "	136°
1.0 " Left	"	0.5 " Left	136°
0.5 " "	"	0.0 " "	136°

Position of Suspension	Correction	Position of Centre of Vertical Vane	Pendulum ~~Deflections.~~ Readings
2.0 cms. Left	-0.5 cms.	1.5 cms. Left	121°
2.8 " "	do.	2.3 " "	99°
4.0 " "	"	3.5 " "	70°
5.0 " "	"	4.5 " "	46°
7.0 " "	"	6.5 " "	17°
9.0 " "	"	8.5 " "	8°
11.0 " "	"	10.5 " "	2°
Reversed	Pendulum. Vertical	Vane to left of	Suspension
1.4 cms. Right	+ 0.5 cms.	0.9 cms. Right	137°
0.9 " "	do.	0,4 " "	142°
0.5 " Left	"	1.0 " Left	127°
0.5 " Right	"	0	142°
0	"	0.5 " "	136°
1.4 " Left	"	1.9 " "	105°
0.1 " Right	"	0.4 " "	137°
1.0 " "	"	0.5 " Right	140°
0.5 " "	"	0	141°
1.5 " "	"	1.0 " "	143°
2.0 " "	"	1.5 " "	13[illegible]°
1.5 " "	"	1.0 " "	143°
3.0 " "	"	2.5 " "	105°
4.0 " "	"	3.5 " "	78°
5.0 " "	"	4.5 " "	40°
6.0 " "	"	5.5 " "	20°
8.0 " "	"	7.5 " "	0°
17.0 " "	"	16.5 " "	0°
51.0 " "	"	30.5 " "	0°

A vertical section taken under the same conditions as the above horizontal section, namely at a distance of seventy-three centimetres from the transmitter with an oscillating voltage of 1000 volts and a frequency of 130,000 cycles per second impressed on the transmitter, is given in Table V and the results are plotted in figure 20. The position of the centre of the vane was determined by a centimetre scale on the rod R (Fig. 3). When this scale reading was zero the pendulum was at the maximum point of the beam.

(The abscissae in figure 20 and in all the following beam sections represent the position of the centre of the pendulum vane while the ordinates represent the pendulum ~~deflections~~ readings.)

TABLE V.

Vertical Section of Beam at 73 cms. from Transmitter.

Voltage on Transmitter:- 1000 volts.

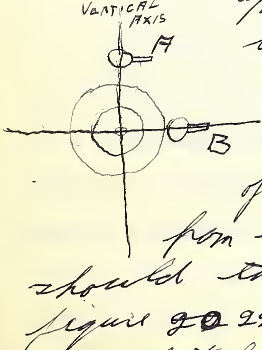
Frequency:- 130,000 cycles per second.

0.002" P.B.S. Suspension 78 cms. long.

Vertical Scale	Pendulum Reading ~~Deflection~~	Vertical Scale	Pendulum ~~Deflection~~ Reading
0.6 cms. Down	136°	8.9 cms. Up	3°
1.6 " "	124°	10.4 " "	0°
2.6 " "	100°	13.9 " "	3°
4.0 " "	47°	15.4 " "	2°
5.6 " "	14°	12.4 " "	0°
6.6 " "	1°	18.4 " "	0°
8.6 " "	0°	26.4 " "	0°
10.1 " "	2°	36.4 " "	0°
11.1 " "	3°	41.4 " "	0°

*

Addendum; – Certain discrepancies between the vertical and horizontal sections are to be expected, due to different orientation of the pendulum. At A the horizontal vane is in a stronger field than it is at B, ~~the~~



the centre of the vertical vane of the pendulum being at equal distances from the centre of the beam. The deflection at A should therefore be ~~greater~~ less than at B. In figure ~~20~~ 22 the vertical sections shows deflections somewhat larger than the horizontal one. Evidently other factors, probably variations in the transmitter, more than counterbalance the effect mentioned above.

00.4 " Up	133°	5.9 " "	16°
1.4 " "	121°	7.8 " "	6°
2.4 " "	97°		

A comparison of figures 19 and 20 shows that the vertical section of the beam, although not identical with, is very similar * to the horizontal one in shape and width, and therefore that the right section of the beam is nearly circular,-as it should be from a circular shaped transmitter. This circular formation is again evident when the horizontal and vertical sections taken at a frequency of 135,000 cycles per second are considered. These sections are plotted in curves one and two of figure 22. It was thought unnecessary to take any more vertical sections as there is no reason to believe that any vertical section would differ materially from its corresponding horizontal section.

The above beam sections served to determine the experimental procedure necessary to obtain reliable surveys of the ultra-sonic beam. A series of experiments was undertaken to investigate the shape of the beam at various ultra-sonic frequencies and at varying distances from the transmitter. The different beam sections taken are tabulated in table VI together with the table and figures in which the data relating to each section is quoted. The method of procedure in all these experiments was exactly the same as that outlined above for the beam at a frequency of 130,000 cycles per second.

TABLE VI

No.	Frequency	Distance from Transmitter	Table No.	Fig[illegible]
1	135,000 cycles per sec.	73 cms.	VIII	22 [illegible] 1
2	do.	do (vertical)	IX	22 " 2
3	"	170 cms	X	23
4	"	300 "	XI	24
5	75,000 " "	73 "	XII	25 " 1
6	45,000 " "	73 "	XIII	25 " 2
7	75,000 " "	202 "	XIV	26 " 1
8	45,000 " "	166 "	XV	26 " 2
9	160,000 " "	73 "	XVI	27

Before proceeding to tabulate the data for the beam sections mentioned in table VI it was thought advisable to plot a few of these beams in polar, instead of cartesian, coordinates as the polar diagrams enable the reader to visualize the "beam formation" of the energy field. The angular displacement of the pendulum from the central axis was plotted as the angular coordinate θ. This angular displacement of the pendulum was obtained by dividing its linear displacement, the abscissae of figures 22 et seg., by the distance from the transmitter at which the section under consideration was taken. The ultra-sonic energy intensities were represented by the vector coordinate. Instead of expressing these intensities as pendulum ~~deflections~~ readings they were expressed as fractions of the maximum intensity which occurred in the section. This method of plotting the beam section will be referred to again in subsection (diii) below.

Three of the sections mentioned in table VI (viz. nos. 1, 5, and 6) were plotted in polar coordinates. The values of the ultra-sonic intensity at various values of θ are quoted in table VII and these results are plotted in figure 21. In this figure the beam formation of the ultra-sonic energy is definitely shown.

TABLE VII

Distance from transmitter = 73 cms.

θ			
155,000 /sec.	75,000 /sec.	45,000 /sec.	Intensity
.101 radians	.195 radians	.254 radians	0
.070 "	.130 "	.186 "	0.1
.061 "	.103 "	.140 "	0.25
.040 "	.075 "	.097 "	0.50
.027 "	.044 "	.060 "	0.75
.016 "	.025 "	.038 "	0.90
0	0	0	1.0

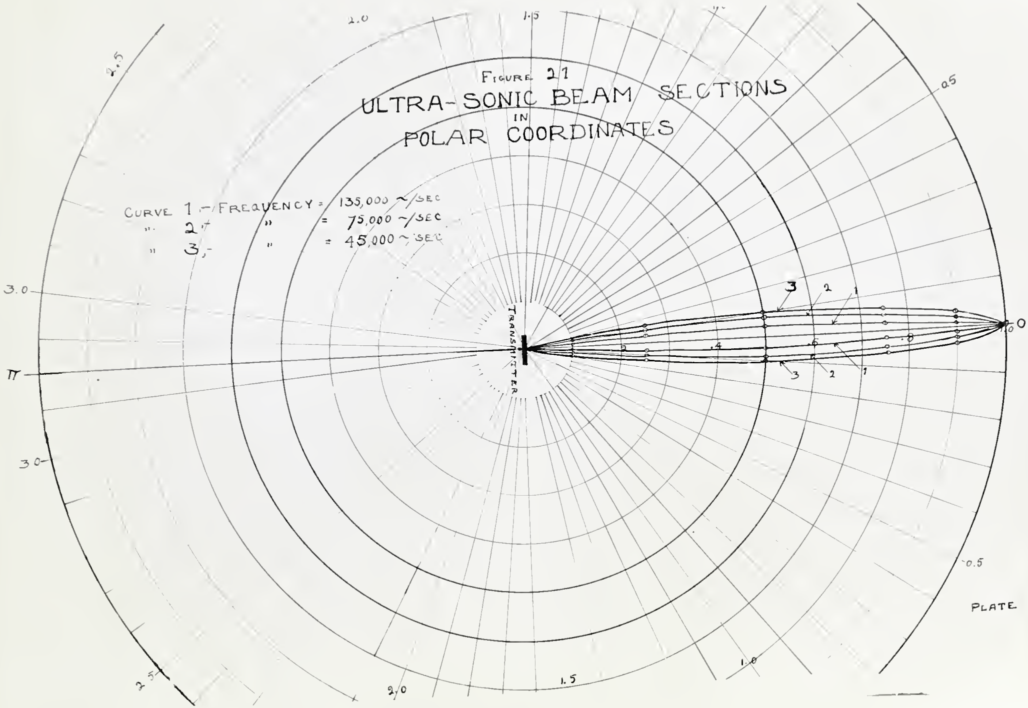


Figure 21
ULTRA-SONIC BEAM SECTIONS IN POLAR COORDINATES

Plate 12

TABLE VIII

Centre of tank at scale reading ~~50~~ 0 cms.
Frequency = 135,000 cycles per second
Horizontal Beam section at 73 cms. from transmitter
Voltage = 1600 volts
Lead Pendulum with circular vanes 1 cm. in diameter.
Suspension,- .0025 inch phosphor bronze strip 78 cms. long.

Position of Suspension.	Correction	Position of Centre of Vert. Vane.	~~Deflection~~ Reading
VERTICAL VANE OF PENDULUM TO LEFT OF SUSPENSION			
1.5 cms. right	0.5 cms.	1.0 cms. right	250°
8.0 " "	do.	7.5 " "	0°
12.85 " "	"	12.35 " "	0°
6.8 " "	"	6.3 " "	11°
7.4 " "	"	6.9 " "	7°
5.75 " "	"	5.25 " "	37°
4.7 " "	"	4.2 " "	97°
9.1 " "	"	8.6 " "	7°
10.1 " "	"	9.6 " "	8°
11.0 " "	"	10.5 " "	5°
11.95 " "	"	11.45 " "	2°
13.9 " "	"	13.4 " "	0
16.0 " "	"	15.5 " "	4°
14.9 " "	"	14.4 " "	2°
17.0 " "	+ "	16.9 " "	4°
17.95 " "	"	17.48 " "	2°
19.0 " "	"	18.5 " "	0°
20.0 " "	"	19.5 " "	0°
3.0 " "	"	2.5 " "	207°
2.35 " "	"	1.85 " "	272°
1.95 " "	"	1.45 " "	315°
1.30 " "	"	0.8 " "	330°
0.65 " "	"	0.15 " "	340°
0.25 " left	"	0.75 " left	302°
1.35 " "	"	1.85 " "	234°
PENDULUM REVERSED			
VERTICAL VANE TO RIGHT OF SUSPENSION			
1.85 " right	-0.5 cms.	2.35 " right	157°
0.75 " "	do.	1.25 " "	235°
0.6 " left	"	0.1 " left	294°
1.65 " "	"	1.15 " "	300°
2.9 " "	"	2.4 " "	172°
5.1 " "	"	4.6 " "	54°
6.05 " "	"	5.55 " "	19°
7.05 " "	"	6.55 " "	8°

TABLE VIII (contin.)

Position at Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Reading
8.95 cms. left	- 0.5 cms.	7.95 cms. left	12°
9.85 " "	do.	9.45 " "	16°
10.95 " "	"	11.45 " "	4°
12.45 " "	"	11.95 " "	0
13.95 " "	"	13.45 " "	2°
15.5 " "	"	15.0 " "	2°
17.05 " "	"	16.55 " "	2°
18.5 " "	"	18.0 " "	0
20.3 " "	"	19.8 " "	0

TABLE IX

Beam Section (Vertical) at 73 cms. from the Transmitter.
Voltage-- 1600 volts.
Frequency--- 135,000 cycles per second
Pendulum, Lead with 1 cm. circular vanes.
Suspension--- .0025 P.B.S. 77.5 cms. long.

Vertical Scale	Reading ~~Deflection~~
3 cms. Up	175°
2 " "	264°
1 " "	333°
0	354°
1 " Down	330°
3 " "	187°
5 " "	50°
7 " "	13°
9 " "	14.5°
11 " "	11°
13 " "	6°
15 " "	4°
17 " "	0
19 " "	0
12 " "	6°
5 " Up	44°
7 " "	8°
9 " "	3°
11 " "	0
13 " "	0
17 " "	0
21 " "	0
10 " "	5°

TABLE X
(Centre of Tank at Scale Reading ~~50~~ 0 cms.)

Beam Section at 170 cms. from the Transmitter
Frequency -- 135,000 cycles per second
Voltage -- 1800 volts
Pendulum, Lead with 1 cm. circular vanes
Suspension, .002 P.B.S. 80 cms. long

Position of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Reading
	VERTICAL VANE TO LEFT		
2.5 cms. Left	0.5 cms.	3.0 cms. Left	126°
0.5 " "	do.	1.0 " "	139°
1.5 " Right	"	1.0 " Right	136°
3.5 " "	"	3.0 " "	117°
5.5 " "	"	5.0 " "	90°
7.5 " "	"	7.0 " "	58°
9.5 " "	"	9.0 " "	26°
11.5 " "	"	11.0 " "	3
13.5 " "	"	13.0 " "	0
	PENDULUM REVERSED, WEST	VANE TO RIGHT	
1.5 " "	0.5 cms.	2.0 cms. Right	124°
0.5 " Left	do.	0.0	133°
2.5 " "	"	2.0 " Left	125°
4.5 " "	"	4.0 " "	100°
6.5 " "	"	6.0 " "	72°
8.5 " "	"	8.0 " "	41°
10.5 " "	"	10.0 " "	16°
12.5 " "	"	12.0 " "	4°
14.5 " "	"	14.0 " "	0

Beam Section at 300 cms. from Transmitter
Frequency --- 135,000 cycles per second
Voltage --- 1800 volts
Pendulum, lead with 2 cm. circular vanes
Suspension, .002 P.B.S. 76.4cms. long.
Centre of Tank at Scale Reading 25 cms.

Position of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Reading
4.5 cms. Right	+ 1.0 cms.	3.5 cms. Right	119°
7.5 " "	do	6.5 " "	86°
10.5 " "	"	9.5 " "	60°
13.5 " "	"	12.5 " "	46°
16.5 " "	"	15.5 " "	-10°
19.5 " "	"	18.5 " "	-32°
22.5 " "	"	21.5 " "	-19°
25.5 " "	"	24.5 " "	-19°
27.5 " "	"	26.5 " "	0
18.0 " "	"	17.0 " "	-47°
22.9 " "	"	21.9 " "	43°
15.4 " "	"	14.4 " "	- 7°
14.2 " "	"	13.2 " "	19°
2.0 " Left	"	3.0 " Left	90°
0.5 " Right	"	0.5 " ~~Right~~	95°
5.5 " "	"	4.5 " RIGHT	107°
9.5 " "	"	8.5 " "	49°
11.5 " "	"	10.5 " "	68°
8.4 " "	"	7.4 " "	75°
1.5 " "	"	0.5 " "	80°
3.5 " "	"	2.5 " "	107°

REVERSED PENDULUM
VERTICAL VANE TO RIGHT OF SUSPENSION

Position of Suspension	Correction	Position of Centre of Vert. Vane	Reading
4.75 " "	-1.0 cms.	5.75 " "	98°
1.5 " "	do.	2.5 " "	105°
1.5 " Left	"	0.5 " Left	92°
4.5 " "	"	3.5 " "	107°
7.5 " "	"	6.5 " "	92°
10.5 " "	"	9.5 " "	49°
13.5 " "	"	12.5 " "	0
16.5 " "	"	15.5 " "	5°
19.5 " "	"	18.5 " "	-15°
22.5 " "	"	21.5 " "	-34°
25.5 " "	"	24.5 " "	- 5°
14.5 " "	"	13.5 " "	-10°
28.5 " "	"	27.5 " "	- 5°
31.5 " "	"	30.5 "	0

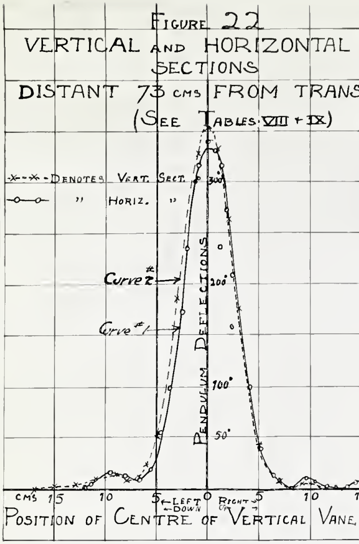


FIGURE 22
VERTICAL AND HORIZONTAL SECTIONS
DISTANT 73 CMS FROM TRANS.
(SEE TABLES VIII + IX)
-x--x- DENOTES VERT. SECT.
-o--o- " HORIZ. "
Curve #2
Curve #1
PENDULUM DEFLECTIONS
300°
200°
100°
50°
CMS 15 10 5 ←LEFT ←DOWN 0 RIGHT→ UP→ 5 10 15
POSITION OF CENTRE OF VERTICAL VANE

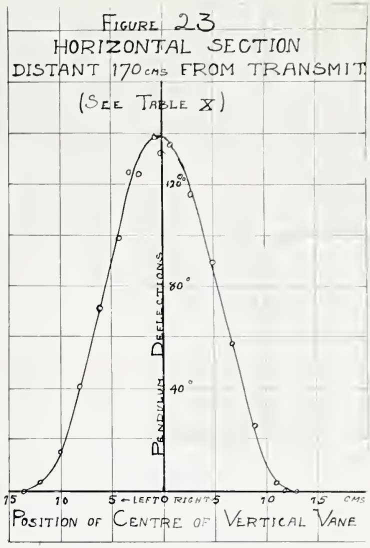


FIGURE 23
HORIZONTAL SECTION
DISTANT 170 CMS FROM TRANSMIT.
(SEE TABLE X)
PENDULUM DEFLECTIONS
120°
80°
40°
15 10 5 ←LEFT 0 RIGHT→ 5 10 15 CMS
POSITION OF CENTRE OF VERTICAL VANE

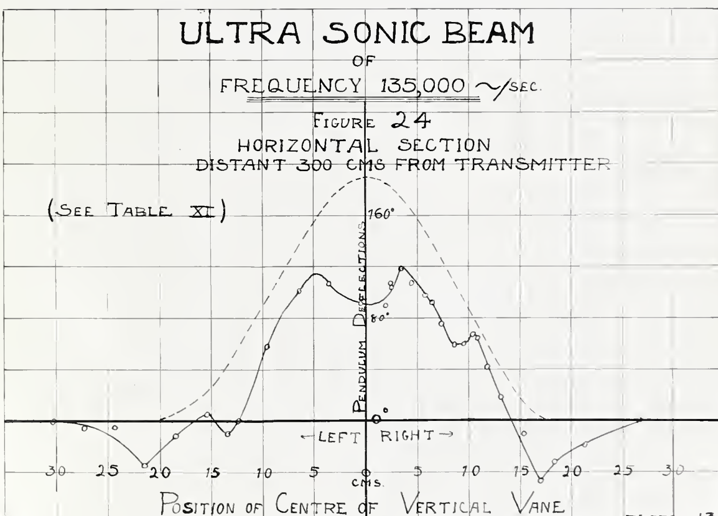

ULTRA SONIC BEAM
OF
FREQUENCY 135,000 ~/SEC.
FIGURE 24
HORIZONTAL SECTION
DISTANT 300 CMS FROM TRANSMITTER
(SEE TABLE XI)
PENDULUM DEFLECTIONS
160°
80°
0°
←LEFT RIGHT→
30 25 20 15 10 5 0 5 10 15 20 25 30
CMS.
POSITION OF CENTRE OF VERTICAL VANE

PLATE 13

TABLE XII

Beam Section at 73 cms. from Transmitter
Frequency --- 75,000 cycles per second
Voltage --- 2,400 volts
Pendulum, Lead with 2 cm. circular vanes.
Suspension, .002 P.B.S. 78 cms long
Centre of Tank at Scale Reading 25 cms.

N.B. The following section was taken 1.4 centimetres below the maximum point of the beam.

Position of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Reading
VERTICAL VANE TO RIGHT OF SUSPENSION			
1.5 cms. Left	-1.0 cms	0.5 cms. Left	182°
3.0 " "	do.	2.0 " "	172°
4.4 " "	"	3.4 " "	147°
7.5 " "	"	6.5 " "	76°
10.5 " "	"	9.5 " "	22°
14.5 " "	"	13.5 " "	3°
0.5 " Right	"	1.5 " Right	170°
0.5 " Left	"	0,5 " "	190°
PENDULUM REVERSED, VERTICAL VANE TO LEFT			
1.5 " "	+1.0 cms.	2.5 " Left	162°
0.5 " Right	do.	0.5 " "	194°
2.5 " "	"	1.5 " Right	189°
5.5 " "	"	4.5 " "	116°
8.5 " "	"	7.5 " "	42°
11.5 " "	"	10.5 " "	9°
13.5 " "	"	12.5 " "	3°

TABLE XIII

Beam Section at 73 cms. from Transmitter
Frequency --- 45,000 cycles per second
Voltage --- 2,700 volts
Pendulum, 3 cms. circular vane
Suspension, .002 P.B.S. 78 cms. long
Centre of Tank at Scale reading 25 cms.

N.B. In this section a second frequency of 3,950 /sec. was present in addition to the 4,500 /sec. as quoted above.

Position of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Reading
VERTICAL VANE TO RIGHT OF SUSPENSION			
1.0 cms. Left	-1.5 cms.	0.5 cms. Right	130°
1.0 " Right	do.	2.5 " "	127°
3.0 " Left	"	1.5 " Left	128°
6.0 " "	"	4.5 " "	100°
11.0 " "	"	9.5 " "	43°
16.0 " "	"	14.5 " "	10°
21.0 " "	"	19.5 " "	0
PENDULUM REVERSED, VERTICAL VANE TO LEFT			
1.0 " "	+1.5	2.5 " "	122°

Position of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ READING
1.0 cms. Right	+ 1.5	0.5 cms. Left	140°
4.0 " "	do.	2.5 " Right	130°
9.0 " "	"	7.5 " "	55°
14.0 " "	"	12.5 " "	16°
29.0 " "	"	17.5 " "	0°

TABLE XIV

Beam Section at 202 cms. from transmitter
Frequency --- 75,000 cycles per second
Voltage --- 2,700 volts.
Pendulum, 2 cms. circular vane
Suspension, .002 P.B.S. 76 cms. long
Centre of tank at Scale reagins 30 cms.

Position of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Readings
0.5 cms. Left	- 1.0 cms	0.5 cms. Right	38°
2.5 " Right	do.	3.5 " "	35°
3.5 " Left	"	2.5 " Left	36°
6.5 " "	"	5.5 " "	29°
9.5 " "	"	8.5 " "	23°
14.5 " "	"	12.5 " "	13°
19.5 " "	"	18.5 " "	6°
22.5 " "	"	21.5 " "	1°
25.5 " "	"	24.5 " "	1°
27.5 " "	"	26.5 " "	-2°
30.5 " "	"	29.5 " "	-5°
35.5 " "	"	34.5 " "	-3°
40.5 " "	"	39.5 " "	-2°
PENDULUM REVERSED, VERTICAL VANE TO LEFT OF SUSPENSION			
2.2 " Left	+ 1.0 cms.	3.2 cms. Left	36°
0.5 " Right	do.	0.5 " "	34°
3.5 " "	"	2.5 " Right	34°
0.5 " "	"	0.5 " Left	35°
2.5 " Left	"	3.5 " "	35°
5.5 " "	"	6.5 " "	29°
0.5 " "	"	1.5 " "	37°
2.5 " Right	"	1.5 " Right	38°
4.5 " "	"	3.5 " "	35°
7.5 " "	"	6.5 " "	31°
12.5 " "	"	11.5 " "	24°
17.5 " "	"	16.5 " "	12°
22.5 " "	"	21.5 " "	5°
27.5 " "	"	26.5 " "	-2°
32.5 " "	"	31.5 " "	-6°
37.5 " "	"	36.5 " "	-4°
42.5 " "	"	41.5 " "	-3°
12.5 " "	"	11.5 " "	20°

TABLE XV

Beam Section at 166 cms. from transmitter
Frequency --- 45,000 cycles per second
Voltage --- 4,000 volts
Pendulum, 3 cms. circular vanes
Suspension, .002 P.B.S. 75.5 cms. long
Centre af tank at scale reading 30 cms.

Position of Suspension	Correction	Position of Centre of Vert. vane	~~Deflection~~ Readings
VERTICAL VANE TO LEFT OF SUSPENSION			
2.0 cms. Right	-1.5 cms.	3.5 cms. Right	55°
3.0 " Left	do.	1.5 " Left	51°
7.0 " Right	"	8.5 " Right	47°
8.0 " Left	"	6.5 " Left	53°
13.0 " "	"	11.5 " "	44°
18.00 " "	"	16.5 " "	33°
23.0 " "	"	21.5 " "	27°
28.0 " "	"	26.5 " "	16°
33.0 " "	"	31.5 " "	8°
38.0 " "	"	36.5 " "	5°
43.0 " "	"	41.5 " "	2°
PENDULUM REVERSED, VERTICAL VANE TO RIGHT OF SUSPENSION			
3.0 cms. Left	+1.5 cms.	4.5 cms. Left	58°
2.0 " Right	do.	0.5 " Right	52°
7.0 " "	"	5.5 " "	59°
17.0 " "	"	15.5 " "	32°
27.0 " "	"	25.5 " "	13°
37.0 " "	"	35.5 " "	2°
42.0 " "	"	40.5 " "	0

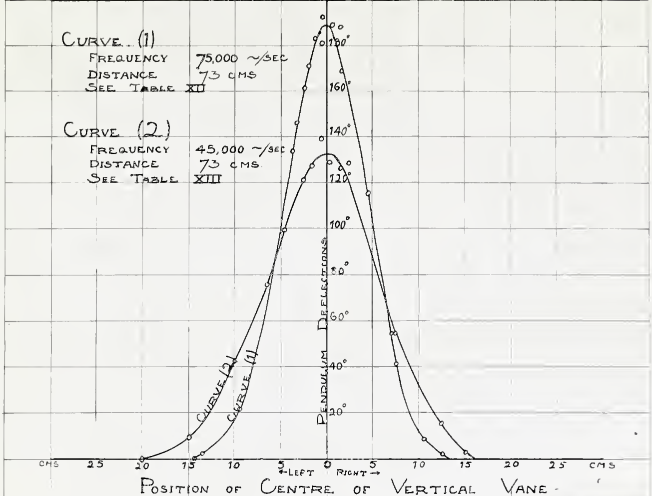
TABLE XVI

Beam Section at 73 cms. from Transmitter
Frequency --- 160,000 cycles per second
Voltage --- 1,300 volts
Pendulum, 1 cm. circular vanes
Suspension, .002 P.B.S. 77 cms. long
Centre of Tank at 30 cms.

Position of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Readings
VERTICAL VANE TO LEFT OF SUSPENSION			
1.4 cms. Right	-0.5 cms.	1.9 cms. Right	56°
1.0 " "	do.	1.5 " "	71

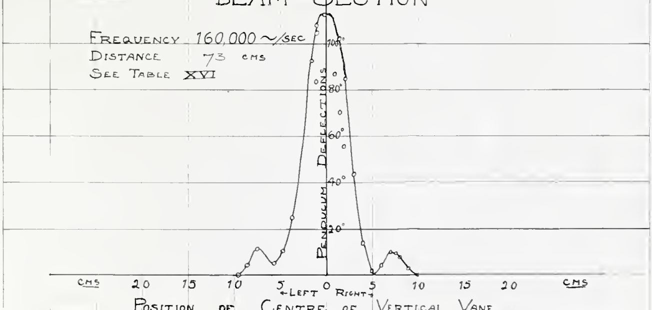
[illegible]

FIGURE 25
ULTRA SONIC BEAM



FOR FIGURE 26 SEE NEXT PLATE

FIGURE 27
ULTRA SONIC BEAM SECTION



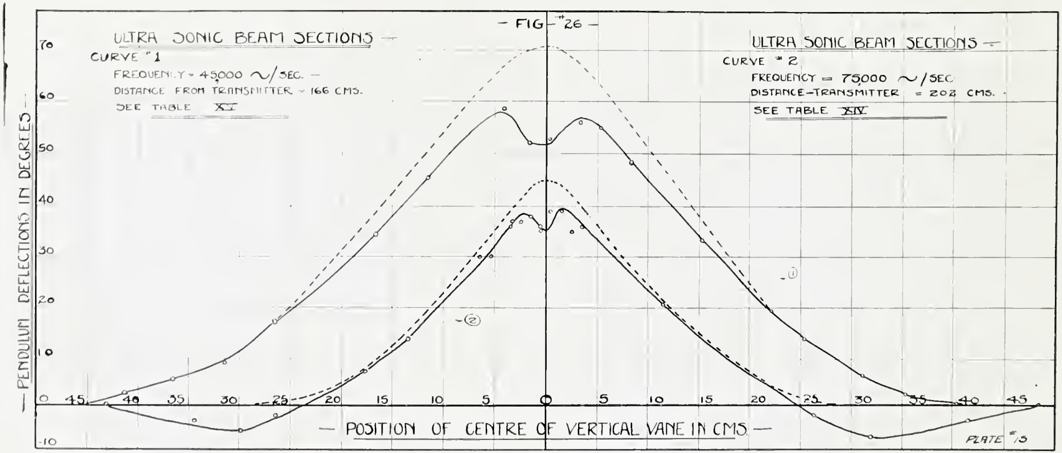

— FIG — #26 —
ULTRA SONIC BEAM SECTIONS —
CURVE #1
FREQUENCY = 45,000 ~/SEC. —
DISTANCE FROM TRANSMITTER = 166 CMS.
SEE TABLE XI
ULTRA SONIC BEAM SECTIONS —
CURVE # 2
FREQUENCY = 75,000 ~/SEC
DISTANCE—TRANSMITTER = 208 CMS.
SEE TABLE XIV
— PENDULUM DEFLECTIONS IN DEGREES —
— POSITION OF CENTRE OF VERTICAL VANE IN CMS —
PLATE #15

TABLE XVI (contin.)

POSITION of Suspension	Correction	Position of Centre of Vert. Vane	~~Deflection~~ Readings
0.5 cms. Right	-0.5 cms.	1.0 cms. Right	87°
0.0 "	do.	0.5 " "	103°
0.5 " Left	"	0.0	108°
1.0 " "	"	0.5 " Left	108°
1.5 " "	"	1.0 " "	105°
2.0 " "	"	1.5 " "	93°
3.0 " "	"	2.5 " "	58°
4.0 " "	"	3.5 " "	25°
5.0 " "	"	4.5 " "	10°
6.0 " "	"	5.5 " "	5°
7.0 " "	"	6.5 " "	11°
8.0 " "	"	7.5 " "	11°
9.1 " "	"	8.6 " "	4°
10.0 " "	"	9.5 " "	0°
REVERSED PENDULUM, VERTICAL VANE TO THE RIGHT			
0.5 cms. Left	+0.5 cms.	1.0 cms. Left	84°
0.5 " Right	do.	0.0	113.5°
1.0 " "	"	0.5 " Right	119°
1.5 " "	"	1.0 " "	120°
2.0 " "	"	1.5 " "	102°
2.5 " "	"	2.0 " "	85.5°
3.5 " "	"	3.0 " "	43.5°
4.5 " "	"	4.0 " "	14.5°
5.5 " "	"	5.0 " "	2.0°
6.5 " "	"	6.0 " "	4.0°
7.5 " "	"	7.0 " "	10.0°
8.5 " "	"	8.0 " "	8.0°
9.5 " "	"	9.0 " "	3.0°
10.5 " "	"	10.0 " "	0

In the above series of experiments the transmitter was placed at one end of the tank. In all the sections taken at distances greater than 100 cms. from the transmitter there was appreciable distortion due to energy reflected from the far end of the tank, the distortion being greater the nearer this end was approached by the pendulum. The greatest distortion occurs in the 135,000 cycle section taken at 300 centimetres from the transmitter. This section was only about 50 cms. from the far end of the tank, in the centre of which there was a glass window. The effect of the window is shown by the humps, a and b, figure 24. Apparently there was more energy reflected from the glass than from the wood and so the central readings are smaller, proportionately, than the outer

readings. A tentative approximate, estimate of the correct section is made in the dotted curves of figures 24 and 26. This estimate was obtained by considering, from the rise and fall of the curve, together with the data quoted in section 5, what would have been, approximately, the maximum pendulum deflection had no reflection been present.

(iii) Conclusions from Above Experimentally Determined Beam Sections

In order to compare the various sections of the different beams it was necessary to express their width in terms of angular distances from a central axis, viz., the axis perpendicular to the face of the transmitter through its centre. This angular distance of any considered point in the beam section was obtained by dividing its linear distance from the axis of the beam by the distance from the transmitter at which the section was taken. It is called θ in the following work and corresponds with the angle θ in Verdet's calculation.

The curves in figures 22, 23, 24, 25, and 26 were replotted on a sheet of rectangular co-ordinate paper but this time the abscissae are the angular distances from the central axis θ and ordinates the energy intensities expressed as fractions of the maximum intensity in the respective section. The curves corresponding to figures 20, and 22 were obtained from the dotted estimate of the correct beam section. These curves are shown in figure 28. In figure 21 the three sections taken at 73 cms. are plotted in polar co-ordinates. These polar curves show very clearly how the energy is emitted from the transmitter in the form of a beam.

The values of intensity I^2 and of the angular distances from the central axis θ, for the different sections, are given in Table XVII.

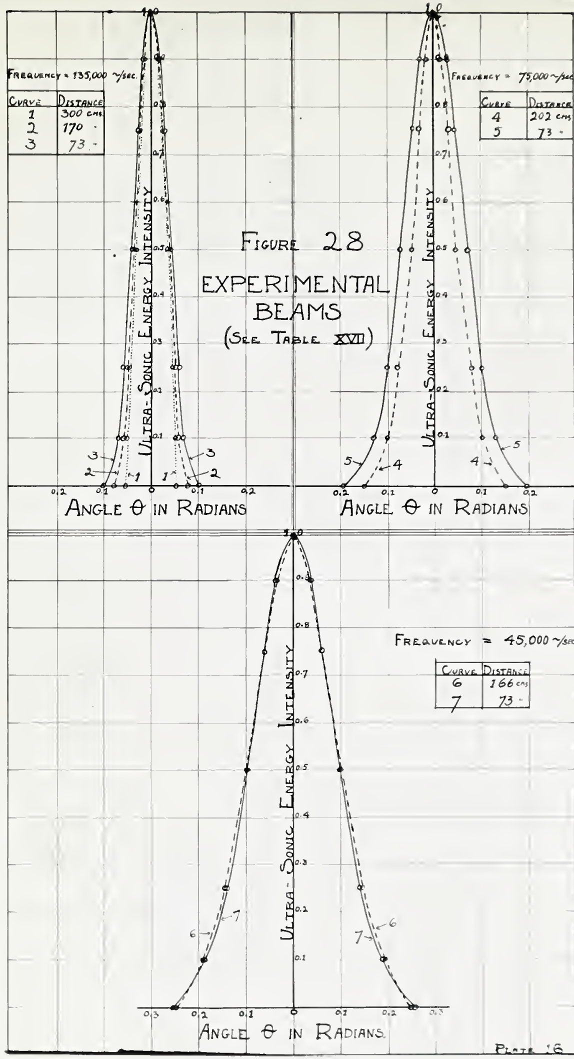


Figure 28
EXPERIMENTAL BEAMS
(See Table XVII)
Frequency = 135,000 ~/sec.
Curve | Distance
1 | 300 cms
2 | 170 "
3 | 73 "
Frequency = 75,000 ~/sec
Curve | Distance
4 | 202 cms
5 | 73 "
Frequency = 45,000 ~/sec
Curve | Distance
6 | 166 cms
7 | 73 "
Ultra-Sonic Energy Intensity
Angle θ in Radians
Plate 16

TABLE XVII

I^2	θ 135,000 ~/sec. 73 cms. from Trans.	135,000 ~/sec. 170 cms. from TRans.	135,000 ~/sec. 300 cms. from Trans.	75,000 ~/sec. 73 cms. from Trans.	75,000 ~/sec. 202 cms. from Trans.	45,000 ~/sec. 73 cms. from Trans.	45,000 ~/sec. 166 cms. from Trans.
0	.101	.077	.064	.195	.151	.254	.248
0.1	.070	.059	.052	.130	.105	.186	.190
0.25	.061	.0494	.043	.103	.081	.140	.144
0.50	.040	.037	.032	.075	.046	.097	.100
0.75	.027	.023	.021	.044	.030	.060	.060
0.90	.016	.014	.013	.025	.015	.038	.036
1.00	0	0	0	0	0	0	0

In figure 28 it will be noticed that as the distance from the transmitter increases, the angular width of the beam decreases. The 25,000 cycles sections do not show this decrease in angular width but when the section at this frequency 73, cms. from the transmitter, was taken, two frequencies were detected by the hertzian wave meter, one of 45,000 cycles and one of 39,500 cycles per second, and for both sections a pendulum with large vanes had to be used. It thus seems probable that these sections are not as reliable as those of higher frequency.

If Verdet's optical equation held exactly the angular width of the beam should remain constant at all distances from the transmitter. It must be remembered, however, that Verdet assumed the points under consideration were at a very great distance from the transmitter. This is not at all the case in the present investigation. ~~A mathematical expression has been developed which shows that, at moderate distances from the transmitter, a slight decrease in the angular width of the beam is to be expected as the distance from the transmitter increases, but that this decrease approaches zero as the distance from the transmitter approaches infinity.~~

~~It has been shown that as the distance from the transmitter approaches infinity the angular width of the beam approaches a constant value.~~ It is the ~~this constant~~ beam at great distances from the transmitter which should be compared with Verdet's calculations rather than the beam obtained from the sections plotted in figure 25. It is interesting, nevertheless, to plot Verdet's results on the same sheet as the experimental section and see just how great a variation does occur. Referring back to subsection (a) we see that according to Verdet:

$$I^2 = K\left(1 - \frac{m^2}{2} + \frac{m^4}{(2!)^2 \cdot 3} - \frac{m^6}{(3!)^2 \cdot 4} \quad \text{etc.}\right)^2$$

Where $m = \frac{\pi R \sin \theta}{\lambda}$

I^2 = Intensity of illumination

K = A constant

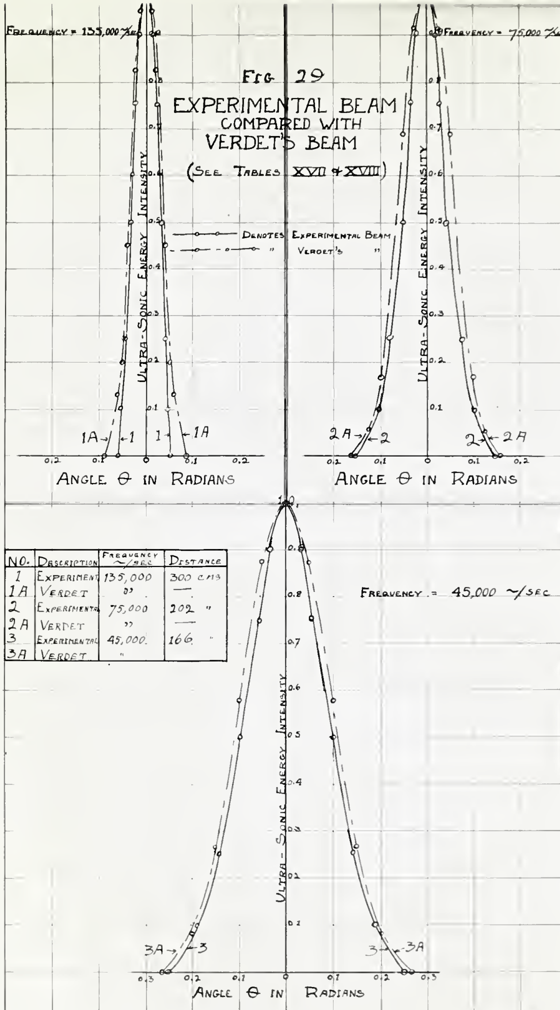
R = Radius of apperture, or, in the ultra-sonic analogy, the radius of the oscillating plate.

θ = Angle of divergence from the central axis

λ = wave length of the energy radiated.

In this expression I^2 represents, in the optical case the intensity of illumination, and corresponds with the ultra-sonic energy density E of section 4. In section (4) it was shown that the energy density was proportional to the pendulum ~~deflection~~ reading, therefore to obtain a comparison between Verdet's optical results and those experimentally determined for the ultra-sonic beam, it is merely necessary to compare Verdet's values of I^2 with the experimental ultra-sonic beams sections of figure 28.

The values of R and λ which occurred in the ultra-sonic sections plotted in figure 25 were substituted in Verdet's equation and the values of I^2 corresponding to different values of θ were calculated. The values of m which made I^2 equal to zero were obtained from Table V. In calculating the ultra-sonic



NO.	DESCRIPTION	FREQUENCY ~/SEC	DISTANCE
1	EXPERIMENT	135,000	300 CMS
1A	VERDET	"	—
2	EXPERIMENTAL	75,000	202 "
2A	VERDET	"	—
3	EXPERIMENTAL	45,000	166 "
3A	VERDET	"	

$\lambda = 1.12$

$\lambda = 2.18$

$\lambda = 3.36$

taken at the greatest distance from the transmitter are used as a comparison with Verdet's results, ~~as~~ for these distant sections are the best available approximation to Verdet's assumption of a very great distance from the source.

Figure 29 shows that Verdet's expression for the optical case is a fair approximation to the experimental ultra-sonic beam. Verdet's beam is somewhat wider than the experimental sections taken at over 100 centimetres from the transmitter and so must be wider than the final ultra-sonic beam a great distance from the transmitter. The discrepancy between Verdet's beam and these experimental ultra-sonic beams might be due to errors in calculating the ultra-sonic wave length. Should the ultra-sonic velocity be less than 1.5 x 10^5 centimetres, as suspected,* the value of λ would be less than that quoted in table XX and Verdet's beams would be narrower. On the other hand, the fact that the discrepancies between Verdet's beam and the experimental beam appear to increase as the frequency decreases would suggest that the effect of the obliquity of the ultra-sonic radiations has to be taken into consideration. As the frequency decreases the width of the beam increases and the effect of obliquity becomes more appreciable. The matter may be investigated further after the ultra-sonic velocity in water has been obtained.

(c) Ultra-Sonic "Side Beams" or Outer Zones.

Referring back to figures 17 and 18 which refer to the sections taken at 73 centimetres from the transmitter, it will be noticed that at a frequency of 135,000 cycles per second, two side beams appear, one on each side of the main beam, while there is a slight indication of a second pair of side beams outside the first pair. These outer zones appear in both the horizontal and vertical sections and therefore are in the form of circular cones of ultra-sonic energy, surrounding the main beam, and separated from it by a zone of zero intensity. The theory of interference, made plain in the work of Verdet and Airy, indicates that these outer zones must exist, but the energy in them is very small compared with the energy of the central beam.

* U/S. Velocity as determined by "standing wave method" = 1.5 x 10^5 cm/sec at °C
July/5/22

In order to obtain more distinct evidence of these outer zones the section was repeated with a more sensitive suspension, the pendulum still being distant 73 centimetres from the transmitter. A suspension of .0015 inches phosphor bronze strip was used. In the main beam the ~~deflections~~ readings were extremely large, much too large to be obtained with any great accuracy, and so were not quantitatively determined. All the readings in the outer portions of the section were negative, showing that energy reflected from the end of the tank was greater than the incident energy in the outer zones. The energy of the outer zones was superimposed on the reflected energy as shown in figure 24. It was almost impossible to obtain the relative intensities of the outer zones for the reflected energy falls off towards the outside of the zone from its maximum intensity at the centre, (see figure 10.)

The numerical results obtained are tabulated in table XIX and plotted in figure 31. The main central beam in figure 31 has a maximum of the order of 3,000 degrees.

TABLE XIX

Beam Section at 73 cms. from transmitter, showing presence of outer zones.

Frequency --- 135,000 cycles per second.

Voltage --- 1600 volts.

Pendulum, circular vanes 1 cm. in diamter

Suspension, .0015 "P.B.S. 78 cms long.

Centre of tank at scale reading 30

VERTICAL VANE TO RIGHT	
Position of Centre of Vertical Vane	Pendulum Reading
12.5 cms. Left	21°
10.5 " "	111°
9.5 " "	130°

TABLE XIX (contin.)

Position of Centre of Vert. Vane	Pendulum ~~Deflection~~ Reading
8.5 cms. Left	133°
9.5 " "	137°
7.5 " "	109°
14.5 " "	-10°
13.5 " "	-17°
12.5 " "	- 1°
11.5 " "	35°
10.5 " "	73°
15.5 " "	-12°
15.0 " "	-13°
16.5 " "	-19°
18.0 " "	-17°
19.0 " "	-19°
30.5 " "	-30°
29.5 " "	-36°
32.5 " "	- 9°
34.5 " "	-14°
32.5 " "	- 8°
31.5 " "	-18°
33.5 " "	-21°
34.5 " "	- 9°
35.5 " "	0
20.5 " "	-22°
22.0 " "	-28°
23.5 " "	-35°
26.0 " "	- 7°
24.5 " "	-47°
23.5 " "	-54°
22.5 " "	-63°
21.5 " "	-47°
20.5 " "	-33°
19.5 " "	-21°
18.5 " "	-31°
18.5 " "	-27°
26.5 " "	-10°
28.5 " "	-35°
27.5 " "	-17°
25.5 " "	-16°
26.5 " "	109°
7.0 " "	93°
5.6 " "	214°
30.5 " "	-29°
VERTICAL VANE TO THE LEFT	
15.0 " Right	- 6°
13.5 " "	14°
13.5 " "	- 5°
at 13.5 a very unsteady condition occurred.	
14.0 cms. Right	- 5°
13.0 " "	-29°
12.5 " "	-23°
10.0 " "	- 1°

TABLE XIX (contin.)

Position of Centre of Vert. Vane	Pendulum ~~Deflection~~ Reading
11.0 cms. Right	32°
8.0 " "	38°
8.5 " "	37°
7.0 " "	11°
6.0 " "	- 2°
5.5 " "	33°
4.5 " "	155°
6.5 " "	- 7°
17.0 " "	-27°
8.0 " "	-20°
19.5 " "	-20°
24.7 " "	-48°
26.5 " "	-32°
28.5 " "	-26°
30.5 " "	-19°
32.5 " "	-10°
34.5 " "	- 8°
22.5 " "	-38°
20.5 " "	-25°
18.8 " "	-19°
16.5 " "	-26°

In order to compare the experimentally determined outer zones with the corresponding zones of Verdet's expression the same method was used as was employed when the main central beam was under consideration. The linear positions of the pendulum for the outer zones in figure 31 were reduced to the corresponding angular distances θ from the central axis, while the intensities were expressed as fractions of the maximum intensity in the main central beam. The intensities of the first outer zone as compared with those of the main beam, were obtained by obtaining the ratio between the maximum intensity of the main central beam and that of the first zone in figure 22, and then determining, from figure 31, the ratio between the first zone and the other outer zones. When obtaining the intensity of the outer zones from figure 31, allowance was made, as indicated before, for the energy reflected from the end of the tank. It was assumed that this reflected energy amounted to about a 90° ~~deflection~~, reading of the pendulum

Figure 31

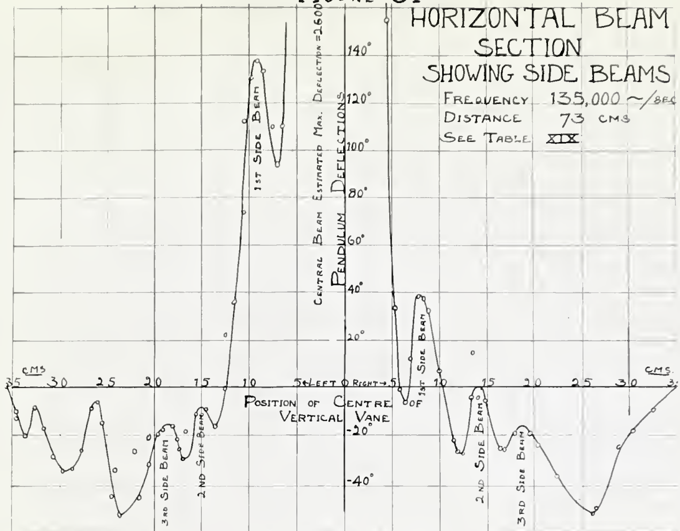
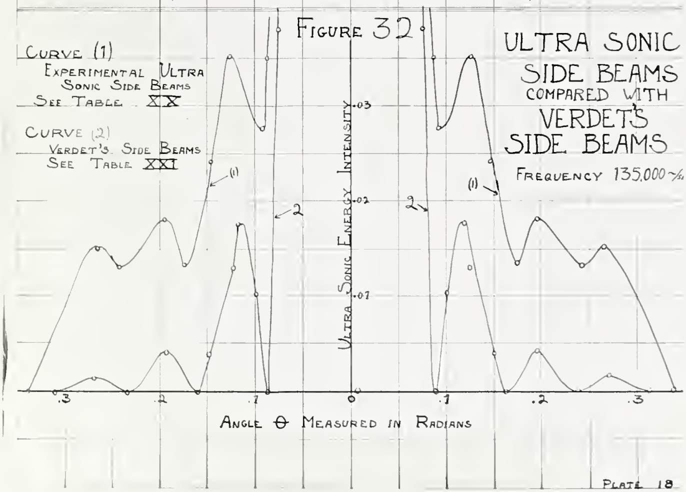


Figure 32



The value of intensity I^2 and angular deflection θ for the experimental outer zones is given in table XX and plotted in curve (1) of figure 31. The values of I^2 and θ for Verdet's optical zones were calculated from the expression:

$$I^2 = K\left(1 - \frac{m^2}{2} + \frac{m^4}{(2!)^2}\,3 - \frac{m^6}{(3!)^2}\,4 \quad etc...\right)^2$$

The results are given in table XXI and plotted in curve (2) of figure 31.

TABLE XX.

I^2	θ	Remarks
.035	.126	Average maximum value of first side beams as obtained from figures 22.
.0275	.0925	Average value of first minimum from fig.31.
.024	.146	~~Average value of second maximum.~~
.0134	.175	Average value of second minimu.
.0180	.195	Maximum of second side beam.
.0130	.243	Third Minimum
.0849	.264	Maximum of third side beam.
about 0	.338	Fourth minimum.

TABLE XXI

Verdet's Optical Outer Zones.

I^2	θ	Remarks
.038	.075	Obtained from figure 25.
0.00	.9885	First minimum.
.0172	.1000	
.0175	.110	Maximum of first side beam.
.0129	.125	
.00407	.150	

TABLE XXI (contin.)

I^2	θ	Remarks
0.00	.163	Second minimum.
.00415	.1945	Maximum of second side beam.
0.00	.237	Third minimum.
.00165	.271	Maximum of third side beam.
0.00	.313	Fourth minimum.

Referring to figure 31 we see that the experimentally determined ultra-sonic side beams coincide fairly closely, as far as their position is concerned, with Verdet's optical side beams, but the intensity of the ultra-sonic beams appears to be greater than Verdet's results would indicate. However, no great reliance can be placed on the experimentally determined intensities because of the presence of a comparatively large amount of reflected energy. But this would not alter the positions of points of maximum and minimum intensity and the agreement between the calculated and the experimentally determined positions of these is good.

Continued in Part II.

ULTRA-SONIC OSCILLATIONS

PARTS II AND III

PART II.

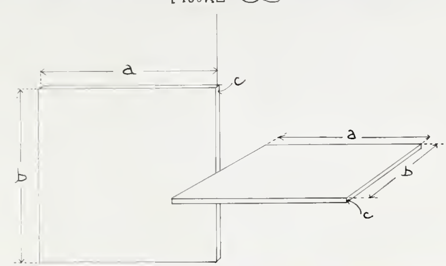
ABSOLUTE MEASUREMENTS OF ULTRA-SONIC ENERGY

In order to determine the absolute value of ultra-sonic energy emmitted by the transmitter, it is necessary to determine the effect of the area and thickness of the pendulum vane upon its deflections, and also the effect of the high frequency voltage applied to the transmitter. A consideration of these questions follows.

Section 8.- Effect of the Area of the Pendulum Vane

In order to determine what effect the area of the vertical vane has on the pendulum deflections, a number of lead pendulums with varying sized square vanes, all of the same thickness, were made and their moments of inertia ~~calculated~~ about their axes of rotation, calculated.

FIGURE 33



In figure30 a sketch of one of these pendulums ~~is given~~. The dimensions a, b, and c are as shown in the figure. If M. represents the mass of the pendulum and I its moment of inertia about its axiis of rotation, then

$$I = \frac{Ma^2}{3} + \frac{Mb^2}{24} + \frac{Mc^2}{24}$$

Let us suppose that uniform incident energy strikes the pendulum and is reflected back directly in its path producing a radiant pressure of ρ dynes per square centimetre. This radiant pressure produces a deflecting torque due to the pressure on the face of the vertical vane on the edge of the vertical vane. If T_v represents the torque on the vertical vane,

T_H the torque on the horizontal vane and T the resultant torque tending to deflect the pendulum, we have

$$T_V = \frac{\rho a^2 b}{2} \quad : \quad T_H = \frac{\rho a^2 c}{2}$$

$$T = T_V - T_H = \frac{\rho a^2}{2}(b - c)$$

This deflecting torque is balanced by the torsion set up in the suspension when the torsion head is turned to bring the pendulum back to the zero position. Let the torsion constant, in dyne cms. per radian of twist, of the suspension be ϕ and the twist of the suspension what we have called "the pendulum ~~deflection~~ reading", be θ radians. Then when the pendulum is brought back to its zero position

$$\phi\theta = \frac{\rho a^2}{2}(b - c) \text{ and } \rho = \frac{2\phi\theta}{a^2(b - c)}$$

The suspension used in one series of experiments was a .003 inch phosphor bronze wire 100 cms. long; in a second series the suspension was of the same wire and 99 cms. long.

The torsion constant ϕ was determined by the oscillation method. For the pendulum used to determine this torsion constant the data was as follows:

a = 4.02 cms.

b = 4.10 cms.

c = 2.73 cms.

M = 101.65 gms.

Whence I = ~~619~~ 648

The period of oscillation for the suspension 100 cms. long was found to be 130 seconds.

therefore ϕ in the first series = ~~1.45~~ 1.51

and ϕ " " second " = 1.47 1.53 ~~1.40~~

These results obtained with the various pendulums when a frequency of 75,000 cycles per second was used are quoted in table XXII and those obtained when the frequency was 135,000 cycles per second are quoted in table XXIII.

TABLE XXII

Frequency --- 75,000 cycles per second.
Voltage --- 2,200 volts.
Suspension , .003" phosphor bronze wire, 100 cms long

Pend. No	a	b	c	ϕ	θ	$P, \frac{\text{dynes}}{\text{cm}^2}$
					Radians	
1	1.95 cms.	2.03 cms.	0.273 cms.	~~1.45~~ 1.51	0.087	~~.0379~~ .0395
2	3.15 "	3.06 "	0.280 "	do.	0.314	~~.0331~~ .0345
3	4.02 "	4.10 "	0.273 "	"	0.58	~~.0273~~ .0287
4	5.95 "	6.02 "	0.281 "	"	1.80	~~.0257~~ .0255
5	8.08 "	8.03 "	0.289 "	"	3.32	~~.0192~~ .0209

TABLE XXIII

Frequency --- 130,000 cycles per second.
Voltage --- 1000 volts
Suspension, .003 phosphor bronze wire, 99 cms long.

Pend. No	a	b	c	ϕ	θ	$P, \frac{\text{dynes}}{\text{cm}^2}$
					Radians	
1	1.95 cms.	2.03 cms.	0.273 cms.	~~1.47~~ 1.53	0.541	~~0.239~~ 0.199
2	3.15 "	3.06 "	0.280 "	do.	1.75	~~0.187~~ 0.195
3	4.02 "	4.10 "	0.273 "	"	2.95	~~0.141~~ 0.147
4	5.95 "	6.02 "	0.281 "	"	5.86	~~0.082~~ 0.085
5	8.08 "	8.03 "	0.289 "	"	8.30	~~0.048~~ 0.050

The above table show that, instead of keeping constant as assumed above, the radiant pressure appears to fall off as the area of the pendulum vane increases; for this there is a natural explanation.

Referring to figures 21 and 15 of section 7 we see that the peak of the 75,000 cycle beam is only one centimetre in width. Therefore, as the area of the pendulum vane increases the outer portions of it are in a weaker field of energy and so the average radiant pressure over the whole vane is less. The fall off in radiant pressure is greater in table XXII than in table XXIII, because the 130,000 cycle beam is narrower than the 75,000 cycle one.

In order to carry out this experiment satisfactorily, one should be sure that the area of the largest vane to be used is less than the area covered by

the peak of the ultra-sonic beam. In other words, a relatively wide beam of low frequency must be used if the pendulum employed for the measurements is a large one. In the work just about to be described we have used a small pendulum, the area of which did not exceed the area of the maximum intensity of the beam. (~~These~~ EXPERIMENTS ON THE EFFECT OF THE AREA OF THE VANE HAVE BEEN SUCCESSFULLY CARRIED OUT SINCE THIS REPORT WAS WRITTEN. SEE PAPER ENTITLED "DIFFRACTIVE SCATTERING OF ULTRA-SONIC ENERGY etc).

9. Effect of the Thickness of the Pendulum Vane.

It has been pointed out that the deflections of a pendulum placed in an ultra-sonic beam are due to the radiation pressure of the ultra-sonic waves, but the magnitude of this radiant pressure depends upon the ratio of the thickness of the pendulum vane to the ultra-sonic wavelength in the material of the pendulum. For this ratio determines the proportion of the incident energy which the pendulum will reflect and, as the radiant pressure on the pendulum is a maximum when the energy incident on it is totally reflected, it follows that the thickness of the pendulum vane has a decided effect upon it deflections.

(a) Mathematical Analysis[1]

Lord Rayleigh has shown that, if a reflecting partition is placed in the path of a plane sound wave, the ratio between the thickness of the partition and the wavelength of the incident energy has a marked effect upon the proportion of energy reflected by the partition. If, in any medium, an incident wave train is represented by

$$\varphi = \varphi' \cos (ax + by + ct)$$

where φ' is the maximum velocity potential in the incident train. Then, if the reflected velocity potential is represented by φ''

$$\frac{\varphi''}{\varphi'} = \frac{\left(\frac{V\rho}{V_1\rho_1} - \frac{V_1\rho_1}{V\rho}\right)}{\sqrt{4\cot^2\frac{2\pi c}{\lambda_1} + \left(\frac{V\rho}{V_1\rho_1} + \frac{V_1\rho_1}{V\rho}\right)^2}}$$

IF WAVE STRIKES PARTITION NORMALLY.

Where:

V = Velocity of sound in water

[1] Lord Rayleigh: Theory of Sound, Vol. 2. page 81

V_1 = Velocity of sound in reflecting partition

ρ = Density of water.

ρ_1 = Density of reflecting partition

λ_1 = Wave length of ultra-sonic vibrations in the reflecting partition.

l = Thickness of reflecting partition.

ϕ' and ϕ'' represent velocity potentials, therefore to obtain <u>intensity</u> relations the square of the above ratio must be taken and we get

$$\frac{\text{Intensity of Energy Reflected}}{\text{Intensity of Incident Energy}} = \frac{\left(\frac{V\rho}{V_1\rho_1} - \frac{V_1\rho_1}{V\rho}\right)^2}{4\cot^2\frac{2\pi l}{\lambda_1} + \left(\frac{V\rho}{V_1\rho_1} + \frac{V_1\rho_1}{V\rho}\right)^2} \quad \text{— (Equation 1)}$$

Let this Ratio = K.

K must have a value between 0 and 1. Substituting the values of V and V_1, ρ and ρ_1 in the above equation, we obtain the relation between K and l/λ. According to the mathematical analysis this relation is independent of the frequency. It depends only on the ratio of l to λ irrespective of the absolute value of λ.

Considering a torsion pendulum we see that the vertical vane acts as a reflecting partition. In applying the above relation to the torsion pendulum it can be deduced that the best thickness for a pendulum wave is one-quarter of a wave length in the material of which the pendulum is composed. The curve showing the relation between K and the ratio l/λ is given in figure 34(1). The pendulums used in obtaining the experimental curve of figure 34(2) were made from lead because the velocity of sound in lead is low and therefore to obtain any required ratio comparatively thin sheets of lead could be used.

If V = Velocity of sound
E = Elasticity of material
ρ_1 = Density of material

Then by Newton's formula

$$V = \sqrt{\frac{E}{\rho_1}}$$

and if n = Ultra-sonic frequency

and λ_1 = Ultra-sonic wave length in the material under consideration

$$\lambda_1 = \frac{1}{n}\sqrt{\frac{E}{\rho_1}}$$

Then if l = thickness of material required to obtain any given value for the ratio l/λ_1. Let this ratio of $l/\lambda = a$

we have $l = a\lambda_1 = \frac{a}{n}\sqrt{\frac{E}{\rho_1}}$

The mass of a pendulum of thickness l would be

$$A\rho_1\sqrt{\frac{E}{\rho_1}} = A\sqrt{E\rho_1}$$

where A is a constant depending of a,n and the area of the face of the pendulum vane.

For convenience in manipulating the apparatus it is desirable to have a pendulum with a quick period of oscillation. This means that the pendulum should have a small moment of inertia and therefore a small mass. The best material to use in making the pendulum, consistent, of course, with other requirements, is one which will give a small value for the product $\sqrt{E\rho_1}$, as when this quantity is small the thickness, and therefore the mass, is also small.

The value of $E\rho_1$ is smaller for lead than for any other material available.

The values of V_1, V, ρ and ρ_1 which in the present case are to be substituted in the equation (1) are

$V = 1.5 \times 10^5$ cms. per second

$V_1 = 2.1 \times 10^5$ " " " (calculated from Newton's formula $V = \sqrt{\frac{E}{\rho_1}}$)

$\rho = 1.0$

$\rho_1 = 11.37$

The values of l/λ_1; $\cot^2\frac{2\pi l}{\lambda_1}$ and K are given in table XXIV, and the values of K and l/λ_1 are plotted in figure 34, curve 1.

TABLE XXIV

l/λ_1	$\boxed{4}\cot^2\frac{2\pi l}{\lambda_1}$	K	l/λ	$\cot^2\frac{2\pi l}{\lambda_1}$	K
0	∞	0	.20	~~1.89~~ .423	~~.965~~ .983
.01	1013	~~.058~~ .0199	.25	0	~~.994~~ .984
.02	252	~~.198~~ .487	.30	~~1.89~~ .423	~~.965~~ .983
.03	112	~~.357~~ .685	.40	~~9.5~~ 7.6	~~.862~~ .958
.04	62.8	~~.497~~ .796	.45	~~39.9~~ 38.1	~~.674~~ .858
.05	~~39.9~~ 38.1	~~.604~~ .858	.47	112.0	~~.357~~ .685
.06	~~27.3~~ 25.3	~~.692~~ .897	.49	1013.0	~~.058~~ .0199
.08	~~15.6~~ 13.3	~~.790~~ .938	.50	∞	0 0
.10	~~9.5~~ 7.6	~~.862~~ .953			

b. Experimental Investigation:

In order to check the theory just given and to get actual experimental data on this question of the thickness of the vane, nine lead pendulums were made with the same area of vane, but of different thicknesses. The diameter of the vanes, circular in ~~shape~~ shape, was four centimetres, while their thickness varied from .0018 cms. to .777 cms. The same suspension was used for them all.

(NOTE: While taking a reading with pendulum 9 the suspension broke and a considerable shorter length had to be used. The correction for the length of suspension in this reading was rather large and as a result this reading is not as reliable as it might be.)

The ultra-sonic frequency used throughout the experiment was 138,000 cycles per sec. Assuming that the ultra-sonic velocity in lead is 2.1×10^5 cms. per second, this frequency would give a wave length in lead of $\frac{2.1}{1.38} = 1.52$ cms. The voltage on the transmitter was 1630 volts and the suspension used was a .0025 inch phosphor bronze wire.

FIGURE 34

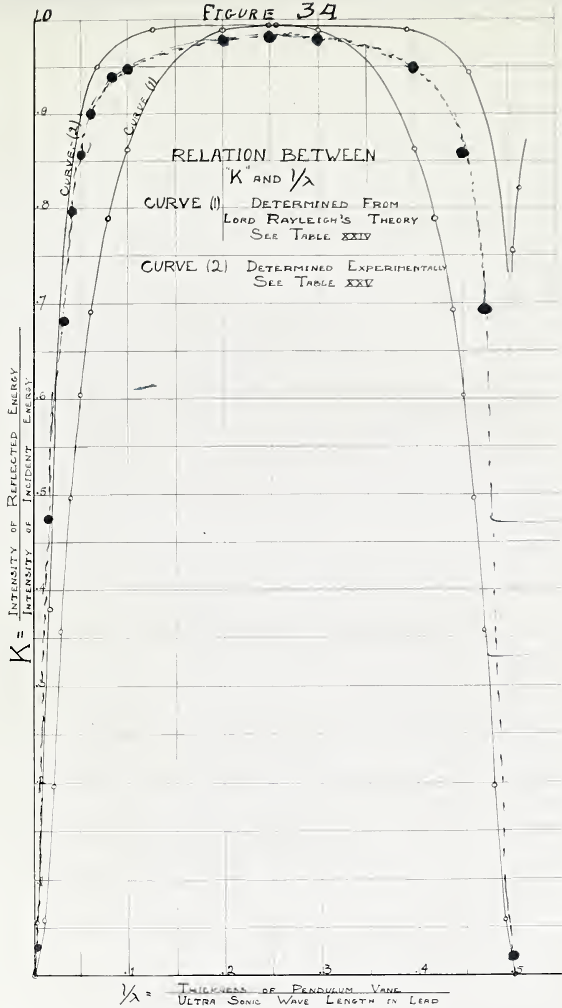


PLATE 19

If the energy incident on the pendulum is E ergs and the ratio of the reflected energy to the incident energy is K, then the reflected energy = KE ergs, and assuming no energy is absorbed, the energy transmitted = (1-K)E ergs. The incident and reflected energy each produce a positive pendulum deflection while the transmitted energy tends to produce a deflection in the opposite direction. If δ = pendulum readings obtained after the pendulum has been returned to its initial position, we have

$$\delta = A(KE + E - (1-K)E)$$

or $\delta = 2AKE$ where A is a proportionality factor ~~correcting~~ CONNECTING the pendulum reading to the energy which produces this reading. We see that, if all other factors remain constant, the pendulum reading will be proportional to K. That is if K_1 and K_2 represent two different values of K and δ_1 and δ_2 the corresponding pendulum readings, then

$$\frac{\delta_1}{\delta_2} = \frac{K_1}{K_2} \qquad (2)$$

In the present experiment the maximum value obtained for δ was 178°. From the ~~experimental~~ theoretical curve figure 34 curve (1) we see that the maximum value of K is 0.994. From equation (2) we see that were K equal to 1, the pendulum reading would be $\frac{178}{0.994} = 179°$.* Therefore to obtain the value of K corresponding to any reading in table XXV it is merely necessary to divide the given reading by 179. The values of K quoted in table XXV were obtained in this way and are plotted against the corresponding values of ℓ/λ_1 in figure 34 curve (2).

* It was assumed that the maximum reflection from the pendulum was as indicated by Rayleigh's expression. In view of the fact that the experimental and theoretical curves do not coincide it is doubtful if this assumption holds. More experimental information on this point is required.

TABLE XXV

Frequency --- 135,000 cycles per second.

Wave length in lead = = 1.52 cms.

Pendulum Number	Thickness of Pendulum	l/λ_1	Length of Suspension	Pendulum ~~Deflection~~ Reading	Corrected Pend. ~~def.~~ Read.	K
1	.0018 cms.	.00118	87.0 cms.	10°	10.0°	.056
2	.024	.0158	86.8	68°	68.4°	.382
3	.102	.067	87.4	170°	170.0°	.950
4	.191	.126	87.2	177°	177.0°	.988
5	.389	.256	87.3	178°	178.0°	.994
6	.599	.394	87.3	177°	177.0°	.988
7	.697	.459	87.8	170°	169.0°	.944
8	.777	.511	87.8	148°	147.0°	.821
9	.765	.504	62.1	96°	135.0°	.755

The theoretical curve of figure 34 curve (1) and the experimental one, figure 34 curve (2) both show that the best thickness for a pendulum reflecting vane is a quarter wave length. There is considerable discrepancy between the two curves. Curve (2) is flatter than curve (1) but a certain similarity can easily be observed. It will require further investigation to find the cause of this discrepancy. Some of these causes may be surmized, as for instance, absorption and scattering of the incident energy by the pendulum. The immediate aim was not to make an exhaustive study of the question, but rather to obtain experimental data as to the effect of the thickness of the vane for use in our measurements.of absolute energy intensity. Figure 34 curve (2) shows that a vane between 0.1 and 0.4 wave lengths thick is almost a perfect* reflector but outside this range the amount of energy reflected falls off very quickly.

* This again assumes that Lord Rayleigh's theoretical value of the maximum reflected energy applies in the present [illegible]

10. Effect of Voltage Applied to Transmitter on the Energy Emission

Theoretically the energy emitted from a transmitter, other things being equal, should vary with the square of the voltage applied to the instrument. It was desired to prove this experimentally. Also, since in any lengthy series of experiments it is practically impossible to keep a voltage absolutely constant, it was necessary to know the relation between the ultra-sonic energy radiated in order to make allowance for the fluctuations. In many of the experiments quoted in this report the voltage fluctuated as much as six percent from its quoted value, although in the later work, when more adequate voltage control was available the fluctuations were confined to within two percent of the quoted voltage.

The ultra-sonic waves are generated by the piezo-electric displacements in the quartz plate of the transmitter. The amplitude of the ultra-sonic wave is therefore proportional to the piezo-electric displacement in the quartz, which in turn, is proportional to the voltage applied to the instrument. Since the ultra-sonic energy is proportional to the square of the amplitude of the wave, we see that the energy emitted should be proportional to the square of the voltage imposed on the transmitter. Before this relation could be used it had to be checked experimentally.

The torsion pendulum was suspended at the position of maximum intensity of the beam at a certain distance from the transmitter. The frequency of electrical oscillations applied to the transmitter was kept constant and the voltage was varied by adjusting the filament heating currents of the thermionic valves. The readings of the pendulum for different voltages are quoted in table XXVI, together with the logarithms to base 10 of these deflections and voltages.

Three series of experiments were taken. Series 1. was taken at a frequency of 45,000 cycles per second; series 2. at a frequency of 75,000 cycles, and series 3. at 135,000 cycles, the last being the resomant frequency of the transmitter.

In all cases the distance from the Transmitter was 73 centimetres.

TABLE XXVI

Series 1. Frequency---45,000 cycles

Voltage	Log_{10} Voltage	Pendulum Reading	Log_{10} Reading
510	2.708	2^{0}	0.301
800	2.903	10^{0}	1.000
1010	3.004	18^{0}	1.255
1310	3.117	32^{0}	1.505
1890	3.253	52.5	1.720
2240	3.350	86^{0}	1.934
2750	3.439	161o	2.207
Series 2		Frequency---75,000 cycles	
810	2.908	10^{0}	1.000
1010	3.004	22^{0}	1.342
2380	3.107	33^{0}	1.519
1590	3.201	50^{0}	1.699
2040	3.310	96^{0}	1.982
2310	3.364	130^{0}	2.114
Series 3		Frequency---135,000 cycles	
1550	3.190	305^{0}	2.483
1210	3.083	215.5^{0}	2.332

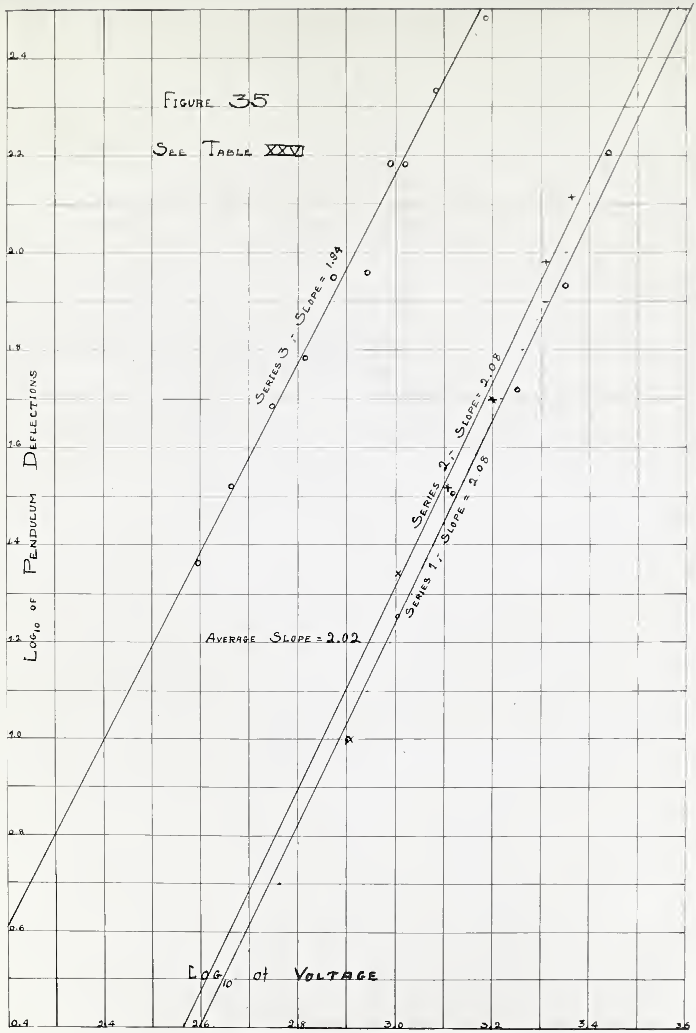


FIGURE 35
SEE TABLE XXVI
SERIES 3 ;- SLOPE = 1.94
SERIES 2 ;- SLOPE = 2.08
SERIES 1 ;- SLOPE = 2.08
AVERAGE SLOPE = 2.02
LOG_{10} OF PENDULUM DEFLECTIONS
LOG_{10} of VOLTAGE
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
2.4
2.6
2.8
3.0
3.2
3.4
3.6

TABLE XXVII (contin.)

Voltage	Log._{10} Voltage	Pendulum ~~Deflection~~ Reading	Log._{10} ~~Deflection~~ Reading
1050	3.021	153^{0}	2.185
880	2.944	91^{0}	1.959
965	2.985	156^{0}	2.193
750	2.875	89^{0}	1.949
395	2.597	23^{0}	1.362
462	2.665	33^{0}	1.519
559	2.747	48.5	1.686
652	2.814	61.0	1.785

The above voltages were measured by electrostatic voltmeters, reading up to 3000 volts calibrated against a standard cell.

In figure 35 the logarithm of the deflection is plotted against the logarithm of the voltage, and the resulting graphs are straight lines with slopes 2.08 2.08 and 1.94. The average slope is therefore 2.02. The greatest deviation from the mean slope is about three percent. This is good agreement considering all the factors of the experiment and we see that, within the limits of experimental error, the ultra-sonic energy emission is proportional to the square of the voltage applied to the transmitter.

11. Absolute Intensity of Ultra-Sonic Energy.

In the above sections we have considered the effect of, first, the area of the pendulum vane; second, the thickness of the pendulum vane; third, the voltage applied to the transmitter; and fourth, in sections 6 and 7, the effect of frequency of the ultra-sonic energy. We shall now proceed to determine the absolute intensity of ultra-sonic energy in the central ultra-sonic beam, taking into consideration the effect of all the above conditions, and the precautions they render necessary.

(a) Mathematical Development:

In section 7, subsection (d) (f), it was pointed out that to obtain correct readings of the energy intensity the integral of the radiant pressure over the face of the transmitter should be taken into consideration. We can now develop an expression to give the relation between the ultra-sonic energy at a given point and the pendulum deflection it produces when the pendulum is suspended at that point.

A section of the torsion pendulum is shown in figure 36.

The following symbols will be used:

x = linear distance of an element of area da of the pendulum from a vertical axis through the point of suspension S.

da = element of area of face of pendulum.

E_0 = intensity of ultra-sonic energy at point of suspension S.

p_0 = radiant pressure of ultra-sonic energy at S.

p = radiant pressure of ultra-sonic energy at da.

Referring to any of the beam sections taken in section 7 of the report we see that p is not constant, but varies with the position of the point under consideration. The value of p at the outer edges of the vanes is different from its value at S. The beam sections obtained also show that over small ranges, such as would be covered by a pendulum vane if it is small, the relation between the radiant pressure and the position of an element of the pendulum vane can be taken as linear, and we have, therefore,

$$p = p_0 + kx$$

where k is the average rate of change of p with respect to x for the small part of the curve ~~considered~~ COVERED BY THE PENDULUM VANE.

In the following we shall let k_v be average rate of change of p over range covered by the vertical vane; and k_h be average rate of change of p over range covered by the horizontal vane.

a_v is radius of the vertical pendulum vane.

a_h is radius of horizontal vane.

Then as the vanes are circular (see figure 36) we can obtain the y coordinate of an element of the vertical vane from the relation

$$y^2 = r^2 - (x - a)^2$$

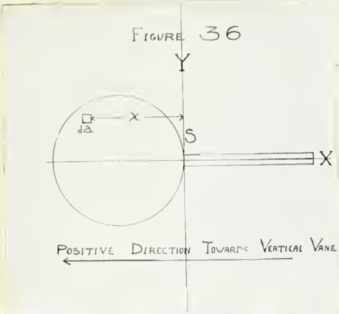


Figure 36

Where r is the radial distance of the element from the centre of the vane.

When r = a we get

$$y^2 = 2ax - x^2$$

$$y = \pm\sqrt{2ax - x^2}$$

The torque due to the radiant pressure on element of area da = $x\,p\,da$ =

$(p_0 + kx)\,x\,da$

where da = dx dy

and the total torque over the vertical vane = T_v

$$T_v = \int_0^{2a_v}\int_{-\sqrt{2ax-x^2}}^{+\sqrt{2ax-x^2}} (p_0 + kx)\,x\,dx\,dy \qquad (1)$$

Taking the positive direction of x as the direction from S towards the vertical vane we get for the horizontal vane a torque T_h

$$T_h = -\int_0^{2a_h}\int_{-\frac{t}{2}}^{+\frac{t}{2}} (p_0 - kx)\,x\,dx\,dy \qquad (2)$$

where t = thickness of horizontal vane.

Integrating T_v with respect to y we get from equation (1)

$$T_v = 2p_0\int_0^{2a_v}\left(\sqrt{2ax-x^2}\right)x\,dx + 2k_v\int_0^{2a_v} x^2\sqrt{2ax-x^2}\,dx \qquad (3)$$

Let $x = 2a_v \sin^2 \theta$

$dx = 4a_v \sin\theta \cos\theta \, d\theta$

and $\sqrt{2a_v x - x^2} = \sqrt{4a_v^2 \sin^2\theta - 4a_v^2 \sin^4\theta}$

$= 2a_v \sin\theta \cos\theta$

also when

$x = 0;\ \theta = 0$ and when $x = 2a_v$ $\theta = \frac{\pi}{2}$

Substituting in (3) we get

$$T_v = 32 \rho_0 a_v^3 \cdot \int_0^{\frac{\pi}{2}} \sin^4\theta \cos^2\theta \, d\theta + 64 k_v a_v^4 \int_0^{\frac{\pi}{2}} \sin^6\theta \cos^2\theta \, d\theta$$

$$\therefore T_v = \pi \rho_0 a_v^3 + \frac{5}{4} \pi k_v a_v^4 \qquad (4)$$

Integrating (2) with respect to y we get

$$T_h = -\rho_0 t \int_0^{2a_h} x \, dx + k_h t \int_0^{2a_h} x^2 \, dx$$

Carrying out this integration we get

$$T_h = -2\rho_0 t a_h^2 + \frac{8}{3} t k_h a_h^3 \qquad (5)$$

The total torque on the pendulum

$T = T_v + T_h$

Adding equations (4) and (5) we get

$$T = \frac{8}{3} t k_h a_h^3 + \frac{5}{4} \pi k_v a_v^4 + \rho_0 \left(\pi a_v^3 - 2 t a_h^2\right)$$

The above torque T was balanced by the torsion of the pendulum suspension and if ϕ represent the torsion ~~comitant~~ CONSTANT of the suspension and θ the twist in the suspension (pendulum ~~deflection~~ reading) required to turn the pendulum to its zero position, measured in radians.

Then the restoring torque is $\phi\vartheta$ and when the pendulum is in its zero position $\phi\vartheta = T$

or $$\phi\theta = \frac{8}{3} t k_h a_h^3 + \frac{5}{4}\pi k_v d_v^4 + p_0(\pi d_v^3 - 2 t d_h^2)$$

and $$p_0 = \frac{\phi\theta - \left[\frac{8}{3} t k_h a_h^3 + \frac{5}{4}\pi k_v d_v^4\right]}{\pi d_v^3 - 2 t d_h^2} \quad \text{dynes}/\text{cm}^2$$

Now $p_0 = 2KE_0$

where E_0 = energy density at the point of suspension

and $K = \dfrac{\text{energy reflected by pendulum}}{\text{energy incident on pendulum}}$

The numerical values of K are given in section 9.

$$E_0 = \frac{\phi\theta - \left[\frac{8}{3} t k_h d_h^3 + \frac{5}{4}\pi k_v d_v^4\right]}{2K\left[\pi d_v^3 - 2 t d_h^2\right]} \qquad (6) \quad \text{Ergs per cubic cm.}$$

The above equation will be known as equation (6) in the following work.

In evaluating the above expression the values of the pendulum ~~deflection~~ reading must be expressed in radian measure and not in degrees.

In developing the expression for T_v and T_h above, no allowance was made for variations in p in a vertical direction. It was assumed that the values of p were constant over the vertical range covered by the ~~suspension~~ VANE. This assumption was justified because the horizontal sections under consideration were taken through the point of maximum intensity of the vertical section of the beam. Figures 20 and 22 of section 7 show that at the peak of the vertical section the radiant pressure is const~~ant~~ANT over the range covered by the vertical pendulum vane

The torsional constant ϕ of the suspension was obtained by the oscillation method. The period of oscillation of the pendulum, in air, was obtained and the torsion constant of the suspension determined from the formula

$$T = 2\pi\sqrt{\frac{I}{\phi}}$$

or

$$\phi = \frac{4\pi^2 I}{T^2}$$

where T = period of oscillation of the pendulum

φ = torsional constant of the suspension

I = moment of inertia of the pendulum.

In order to determine φ the moment of inertia of the pendulum had to be known. This was obtained by determining the period of the pendulum and then attaching a body of known moment of inertia to the pendulum and obtaining the period of the compound body so formed.

If T = period of pendulum alone

T_1 = period of pendulum plus known body

I = moment of inertia of pendulum

I_1 = moment of inertia of known body

φ = torsion constant of suspension

$$T^2 = \frac{4\pi^2 I}{\varphi}$$

$$T_1^2 = \frac{4\pi^2 (I + I_1)}{\varphi}$$

Dividing we get

$$\frac{T_1^2}{T^2} = \frac{I_1 + I}{I} \quad \text{or} \quad \frac{I_1}{I} = \frac{T_1^2 - T^2}{T^2}$$

Therefore $$I = \frac{I_1 T^2}{T_1^2 - T^2}$$

The body of known moment of inertia consisted of a disc suspended by a thin stiff wire from the centre (point of suspension) of the pendulum as shown in figure 36. In table XXIX the dimensions of the different pendulums used in obtaining absolute intensity measurements together with their moments of inertia.

The symbols used in table XXIX are the same as those used in the above mathematical work namely:

a_v = radius of vertical vane

a_h = radius of horizontal vane

t = thickness of vanes

I - moment of inertia

FIGURE 37

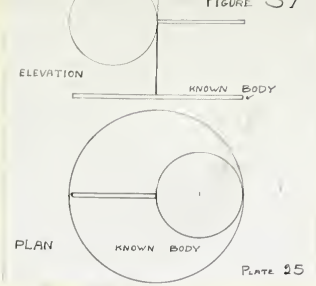


TABLE XXVII

Pendulum Number	a_v	a_h	t	I
1	0.503 cms.	0.505 cms.	0.254	2.44
2	1.000 "	1.000 "	0.239	24.73
3	1.515 "	1.515 "	0.239	138.4

(b) Experimental Procedure

In section 7 we have determined the relative distribution of ultra-sonic energy in the main central beam, and have expressed the energy intensities at different points as fractions of the maximum intensity at the peak of the beam. In order to obtain the absolute intensity at any given point in the same section all that is now required is to determine the absolute intensity on the central axis, i.e. at the peak of the beam.

The vertical position of maximum intensity was first obtained by moving the torsion pendulum up and down in the same way as described in section 7. Through this vertical position of maximum intensity a horizontal section of the peak of the beam was taken.

Most of the work was done at frequency of 135,000 cycles per second, the resonant point of the transmitter, and at ~~thei~~ this frequency sections of the peak of the ultra-sonic beam were taken at four different distances from the transmitter. Sections were also taken at 75,000; ~~76,000~~ 67,000 and 45,000 cycles per second.

From these experimental sections of the peak of the ultra-sonic beam the values of θ, k_v and k_h were obtained and the absolute intensity of the ultra-sonic energy was calculated from the equation (6).

The experimental data obtained is tablulated in tables XXVIII and plotted in figures 37 to 40.

TABLE XXVIII

Absolute Intensity of Ultra-Sonic beam.
Frequency --- 135,000 cycles per second.
Voltage on transmitter --- 1,600 volts.
Distance from transmitter --- 61 ~~5~~ cms.
Suspension --- .0025 P.B.S. 77.5 cms. long
Pendulum #1: a_v = 0.503 CMS
a_h = 0.505 "
t = 0.254 "
Moment of Inertia --- 2.44
Period of oscillation at pendulum T --- 26.4 secs.
Torsional constant of suspension
φ = 0.138
Ultra-Sonic wave length in lead λ = 1.555 CMS
ℓ/λ_1 = 0.163 K (see figure 34 curve 2) = 0.99
Vertical vane of Pendulum to left.

Position of Suspension	θ in Radians.
1.8 cms left	2.31
0.8 " "	4.10
0.1 " RIGHT	5.15
1.0 " right	5.20
2.0 " "	4.17
0.5 " "	5.31

XXIX

TABLE XXXX

Voltage, Frequency, Pendulum and Suspension as in Table XXVIII
Distance from Transmitter ---105 cms. Vertical Vane to Left.

Position of Suspension	θ in Radians
5.0 cms. right	1.43
4.0 " "	1.73
2.85 " "	2.70
2.0 " "	2.69
0.5 " "	2.51
1.0 " "	/ 2.15
0.0 " "	2.08

TABLE XXX

Voltage, Frequency, Pendulum and Suspension as in Table XXVIII

Distance from Transmitter --- 170 cms. Vertical vane to left.

Position of Suspension	θ in Radians
5.5 cms. Right	0.76
4.5 " "	0.85
3.6 " "	0.91
2.5 " "	0.85
1.35 " "	0.715

TABLE XXXI

Frequency, Voltage, Pendulum and Suspension as in table XXX.

Distance from Transmitter --- 217 cms. Vertical Vane to Left.

Position of Suspension	θ in Radians
1.2 cms. left	0.34
0.5 " "	0.39
0.55 " Right	0.52
1.5 " "	0.45
3.0 " "	0.39
1.5 " "	0.45
0.5 " "	0.48
4.0 " "	0.48
6.0 " "	0.52
8.0 " "	0.43

In tables XXIX and XXXI the ultra-sonic energy intensity appears to fall off in the centre of the sections,(see figure 38, curves 2 and 4). At first it was thought that this effect was due to reflected energy which had not been entirely removed by the dissipating screens described in section 5. If this were the case, however, the same effect should be noticeable in table XXX and figure 38, curve 3, as this section was taken at a distance intermediate between the other two sections. In later work with other transmitters (see part III) it was found that slight fluctuations in the frequency of the electrical generating circuit when working at the instrument's resonant point, produced large fluctuations in the pendulum ~~deflection~~ reading. Possibly some such effect is occurring in the present case; but if so it is the only example of such large frequency fluctuations met with during the months of experimental work with this transmitter.

TABLE XXXII

Voltage, Frequency and Pendulum as in table XXVIII

Suspension --- .0025 P.B.S.; 78 cms long

Period of Oscillation T --- 25.8 secs. Torsion Constant

$\phi = 0.145$

Distance from Transmitter --- 73 cms.

(NOTE: Scattering Screens were not present when this series was taken)

The following readings are copied from the peak readings of table VIII, section 7; they are decidedly greater (14%) than the readings in table XXVIII. The readings in table XXVIII were taken two or three months after those given below. It is quite probable that the discrepancies were caused by some change in the condition of the apparatus which occurred during that time. Possibly some deterioration in the transmitting properties of the transmitter occurred; for example, water may have seeped through the protective layers of wax and resin (see section 2) and caused a slight electrical leakage in the instrument.

VERTICAL VANE TO LEFT

Position of Suspension	Radians
3.7 cms right	1.69
2.0 " "	3.62
1.35 " "	4.73
0.95 " "	5.50
0.3 " "	5.76
0.35 " left	5.92
1.25 " "	5.27
2.35 " "	4.10

TABLE XXXIII

The following readings are copied from the peak readings of table XII Section 7.

Frequency --- 75,000 cycles per sec.

Voltage --- 2,400 volts

Pendulum # 2 was used.

A_v - 1.000; a_h - 1.000; t - 0.239; I - 24.73

Period of Oscillation of Pendulum T --- 120.0 sec.

Torsional Constant of Suspension ϕ = 0.0677

Suspension --- .0020" P.B.S.; 78.0 cms. long

Ultra-Sonic Wave Length in Lead λ_1 = 2.80

l/λ_1 = .0855 ; K (see fig. 34) = 0.96

Distance from Transmitter --- 73 cms.

VERTICAL VANE TO LEFT

Position of Suspension	θ in Radians
0.0 cms	2.83
2.0 " Right	3.38
4.0 " "	3.30
7.0 " "	2.03
0.9 " Left	2.56

TABLE XXXIV

The following readings are copied from table XIV, Section 7.

Frequency and Pendulum same as in table XXXIII.

Voltage --- 2,700 volts;

Suspension --- .002 P.B.S. 76 cms. long.

Period of Oscillation T --- 111.4 secs.

Torsion constant. φ = 0.078

Distance from Transmitter --- 202 cms.

VERTICAL VANE TO LEFT

Position of Suspension	θ in Radians
9.0 cms. Right	0.419
4.0 " "	0.541
1.0 " "	0.611
1.0 " Left	0.664
4.0 " "	0.645
3.0 " "	0.611
5.7 " "	0.629
6.0 " "	0.611
9.0 " "	0.506
0.0 " "	0.594

TABLE XXXV

Frequency --- 67,000 cycles per sec. Voltage --- 3,000 volts.

Distance from Transmitter --- 61.0 cms. Pendulum and Suspension as in table XXVIII.

(a)

Position of Suspension	θ in Radians
0.7 cms. Right	0.14
1.0 " Left	0.192

TABLE XXXV (contin.)

Position of Suspension	θ in Radians
2.0 cms. Left	0.229
3.0 " "	0.128
5.0 " "	0.122

(b) Suspension --- .0025 P.B.S. 77.5 cms. long

Period of Oscillation T --- 92.0 cms. $\phi \approx .0677$

Pendulum --- 2 cm. circular vanes

a_v - 1.000; a_h - 1.000; t - .239; I - 24.7

Ultra-Sonic Wave Length in lead λ_1 = 3.13

l/λ_1 = .0765; K (see fig. 34) = 0.94

VERTICAL VANE TO RIGHT

Position of Suspension	θ in Radians
4.0 cms. Left	2.72
6.0 " "	1.40
2.0 " Right	1.59
0.0	2.64
1.0 " Left	3.09
2.0 " "	3.26

TABLE XXXVI

The following readings are copied from table XIII, Section 7.

Frequency --- 45,000 cycles per sec. Voltage --- 2,700 Volts

Distance from Transmitter --- 73 cms. Pendulum --- 3 cms. circ. vanee (#3)

TABLE XXXVI (contin.)

a_v - 1.515; a_h - 1.515; t - 0.239; I - 138.4

Suspension same as in table XXXV ϕ = .0677

Ultra-Sonic wave length in lead λ = 4.66

ℓ/λ = .0518; K (see fig. 34) = .894

VERTICAL VANE TO RIGHT

Position of Suspension	θ in Radians
2.0 cms. Right	2.22
0.0	2.22
2.0 " Left	2.23
5.0 " "	1.74
7.0 " Right	0.96

TABLE XXXVII

Frequency --- 45,000 cycles per sec. Voltage --- 4,000 volts.

Distance from Transmitter --- 130 cms. Pendulum #3 same as table XXXVI

Suspension --- .0020 P.B.S. 75.5 cms. long.

Period of Oscillation T --- 238 secs. Torsion constant of susp.

ϕ = 0.105

VERTICAL VANE TO LEFT

Position of Suspension	θ in Radians
10.2 cms. Right	1.27
5.6 " "	1.67
2.5 " "	1.80
0.3 " "	1.89
2.5 " Left	1.76
5.0 " "	1.59
10.0 " "	1.19

TABLE XXXVIII

The following readings are copied from Table XV , Section 7.
Frequency, Voltage, Pendulum and Suspension as in table XXXVII
Distance from Transmitter --- 166 cms.

VERTICAL VANE TO RIGHT

Position of Suspension	θ in Radians
12.0 cms. Right	0.56
5.0 " "	0.82
0.0	0.96
5 .0 " Left	0.89
10.0 " "	0.92
15.0 " "	0.77
20.0 " "	0.58
25.0 " "	0.47

The results tabulated above are plotted in the following figures:

Readings of Table XXVIII to XXXII plotted in figure 38.
" " " XXXIII " XXXIV " " " 39.
" " " XXXV " " " " 40.
" " " XXXVI " XXXVIII " " " 41.

From the curves given in figures 38 to 41, we can obtain the values of θ; k_v and k_h which when substituted in equation 6. will give the absolute intensity of ultra-sonic energy at the point under consideration. Referring to equation 6. we see that k represents $\frac{\partial p}{\partial x}$ and it was assumed that the value of k was approximately constant over the range covered by the pendulum vanes. Before we can apply equation 6. we require to find the relation between $\frac{\partial p}{\partial x}$ and $\frac{\partial \theta}{\partial x}$ and the

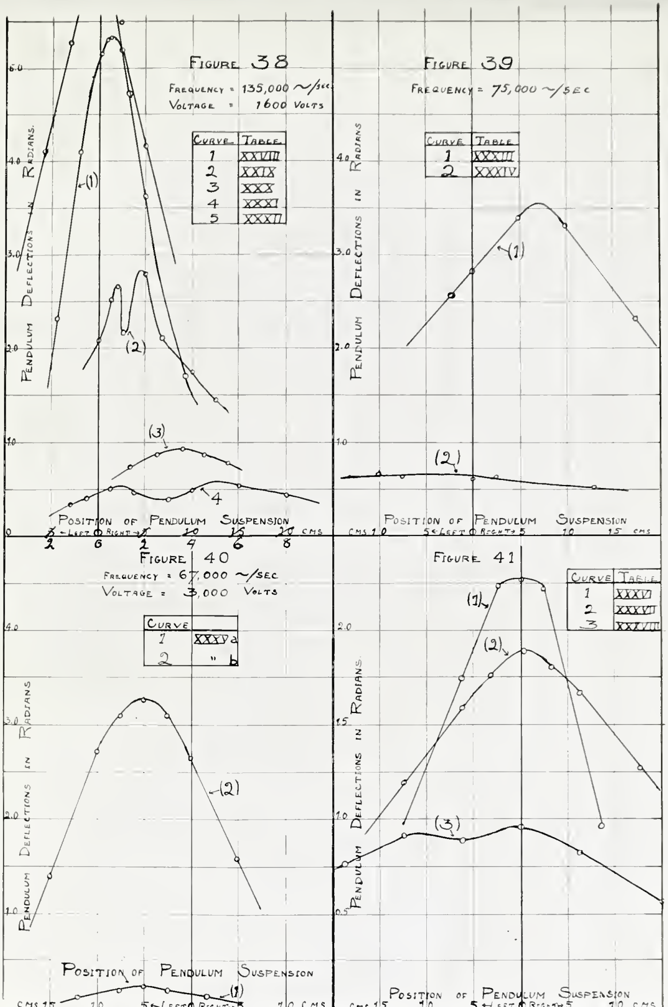


FIGURE 38
FREQUENCY = 135,000 ~/SEC
VOLTAGE = 1600 VOLTS
CURVE TABLE
1 XXVIII
2 XXIX
3 XXX
4 XXXI
5 XXXII
PENDULUM DEFLECTIONS IN RADIANS.
POSITION OF PENDULUM SUSPENSION
← LEFT RIGHT →
CMS
FIGURE 39
FREQUENCY = 75,000 ~/SEC
CURVE TABLE
1 XXXIII
2 XXXIV
PENDULUM DEFLECTIONS IN RADIANS
POSITION OF PENDULUM SUSPENSION
FIGURE 40
FREQUENCY = 67,000 ~/SEC
VOLTAGE = 3,000 VOLTS
CURVE
1 XXXV a
2 " b
PENDULUM DEFLECTIONS IN RADIANS
POSITION OF PENDULUM SUSPENSION
FIGURE 41
CURVE TABLE
1 XXXVI
2 XXXVII
3 XXXVIII
PENDULUM DEFLECTIONS IN RADIANS
POSITION OF PENDULUM SUSPENSION

PLATE 21

value of $\frac{d\theta}{dx}$ may be obtained from figures ~~39~~ to ~~40~~.41

Let $k_v = mk_h$ where m is a constant, For as we have assumed that k_v and k_h are each constant then ~~this~~ k_v must equal a constant times k_h

Now $P_0 = 2KE_0$

So multiplying both sides of equation 6 by 2K we get

$$P_0 = \frac{\phi\theta}{\pi a_v^3 - 2t a_h^2} - \frac{k_v\left[\frac{8}{3}\frac{t a_h^3}{m} + \frac{5}{4}\pi a_v^4\right]}{\pi a_v^3 - 2t a_h^2} \quad \frac{\text{dynes. per}}{\text{sq. cm.}}$$

and differentiating, with respect to x we get

$$\frac{dP_0}{dx} = \frac{\phi}{\pi a_v^3 - 2t a_h^2} \cdot \frac{d\theta}{dx}$$

The second term is zero because k_v was assumed to be constant and if s_v = average value of $\frac{d\theta}{dx}$ over the range covered by the vertical vane and s_h = average value of $\frac{d\theta}{dx}$ over the range covered by the horizontal vane, then

$$k_v = \frac{\phi}{\pi a_v^3 - 2t a_h^2} s_v \quad \text{and } k_h = \left(\frac{\phi}{\pi a_v^3 - 2t a_h^2}\right) s_h$$

The values of θ; s_v and s_h obtained from figure 38, together with the corresponding values of k_v; k_h; and E are given in table XXXIX while the results obtained from figures 39, 40, and 41 are given in tables XL and XLII. In figures 42 to 45 the values of the energy density E are plotted against the corresponding positions of the pendulum suspensions:

TABLE XXXIX

Frequency --- 135,000 cycles; Voltage --- 1,600 volts; Pendulum #1

a_v - .503; a_h - .505; t - .254; Suspension --- .0025 P.B.S . 77 cms.long.

ϕ - 0.138)
) See Table XXVIII
K - 0.99)

Position of Suspension	θ RADIANS	s_v	k_v	s_h	k_h	E_c
Series (1)	Distance from Transmitter --- 61.3 cms.					Ergs per cubic cm.
2.5 cms. Right	3.67 rad.	0.98	0.499	1.01	0.515	0.623
1.5 " "	4.65 "	0.66	0.339	0.98	0.499	0.956
0.5 " "	5.31 "	-0.79	-0.403	0.66	0.339	1.440
0.5 " Left	4.52	-1.72	-0.876	-0.79	-0.403	1.600
1.5 " "	2.80	-1.81	-0.921	-1.76	-0.876	1.292
2.0 " "	1.90	-1.70	-0.866	-1.82	-0.926	1.049
Series (2) See Table XXXI & figure 37 (2). Distance from Transmitter 105 cms						
3.0 cms. Right	2.06	0.64	0.326	0.350	0.178	0.320
2.0 " "	2.70	-0.50	-0.255	0.640	0.326	0.765*
1.3 " "	2.04	-0.36	-0.184	-1.460	-0.234	0.470*
0.7 " "	2.60	-0.80	-0.470	0	0	0.861*
0.5 " Left	1.74	-0.78	-0.398	-0.75	-0.382	0.696
1.5 " "	1.00	-0.78	-0.398	-0.78	-0.398	0.510
Series (3) See Table XXXII and figure 37(3). Distance from Transmitter 170 cms						
6.0 cms. Right	0.68	0.15	0.0765	0.15	0.0765	0.127
5.0 " "	0.83	0.10	0.0510	0.15	0.0765	0.178
3.8 " "	0.93	-0.05	-0.255	0.07	0.0356	0.2455
2.0 " "	0.80	-0.12	-0.612	-0.10	-0.0510	0.2450
1.0 " "	0.68	-0.11	-0.561	-0.12	-0.0612	0.2120
Series (4) See Table XXXIII and Figure 37(4). Distance from transmitter 217 c						
7.0 cms. Right	0.47	0.06	0.0306	0.06	0.0306	0.102*
5.2 " "	0.56	-0.06	-0.0306	0.09	0.0204	0.155*
4.0 " "	0.48	-0.09	-0.0459	-0.08	-0.0408	0.153*
3.0 " "	0.38	+0.03	-0.0153	-0.09	-0.0459	0.100*
1.7 " "	0.49	0.06	-0.306	-0.05	-0.0255	0.095*
0.5 " "	0.50	-0.08	-0.0408	-0.05	-0.0255	0.149*
1.0 " Left	0.36	-0.12	-0.0612	-0.10	-0.0510	0.130
2.0 " "	0.23	-0.12	-0.0612	-0.12	-0.0612	0.099

* The fall off in intensity noted in Tables XXIX and XXXI is again in evidence.

In order to get an estimate of the probable maximum intensity of the beam curves (2) and (4) figure 42 were produced as shown by the dotted line.

TABLE XXXIX (contin.)

Series (5) See Table XXXII and figure 38, curve (5).
Pendulum Suspension changed ϕ = 0.145
Distance from Transmitter --- 73 cms.

Position of Suspension	θ Radians	s_v	k_v	s_h	k_h	E Ergs per cubic cm
2.0 cms. Right	3.89	1.22	0.653	1.14	0.610	0.635
1.0 " "	5.06	0.86	0.460	1.22	0.653	1.050
0.0	5.92	-0.36	-0.193	0.86	0.462	1.620
1.0 " Left	5.48	-0.94	-0.504	-0.37	-0.198	1.750
1.5 " "	5.04	-1.08	-0.578	-0.82	-0.439	1.710
2.5 " "	3.96	-1.12	-0.600	-1.08	-0.578	1.450
3.5 " "	2.84	-1.12	-0.601	-1.12	-0.600	1.180

TABLE XL

Frequency --- 75,000 cycles per sec.; Pendulum #2 a_v = 1.0; a_h = 1.0
t = 0.239; K = 0.96

Series (1) See Table XXXIII & figure 39, curve (1)
Voltage on Transmitter --- 2,400 volts; Dist. from Transmitter --- 73 cms.
ϕ = 0.0677

Position of Suspension	θ Radians	s_v	k_v	s_h	k_h	E Ergs per cubic cm
7.0 cms. Right	2.030	0.43	0.01091	0.40	0.0102	0.0172
5.0 " "	2.880	0.32	0.00814	0.43	0.01091	0.0306
3.0 " "	3.52	-0.19	-0.0048	-0.32	0.00814	0.0486
1.0 " "	3.14	-0.30	-0.0076	-0.19	-0.0048	0.0480
1.0 " Left	2.54	-0.30	-0.0076	-0.30	-0.0076	0.0405
Series (2) See Table XXXIV & Figure 39 (2), Voltage --- 2,700 volts; ϕ = 0.0788 Distance from Transmitter --- 202 cms.						
4.0 cms. Right	0.54	0.004	0.00012	0.003	0.00009	0.0082
0.6 " Left	0.66	-0.001	-0.00003	0.003	0.00009	0.0102
4.0 " "	0.63	-0.001	-0.00003	0.001	-0.00003	0.0097

Table XLI

See Table XXXV & figure 40 (2); Frequency --- 67,000 cycles per second.
Voltage on Transmitter --- 3,000 volts; Pendulum # 3.
a_v = 1.515; a_h = 1.515; t = 0.239; ϕ = 0.115; K = 0.94
Distance from Transmitter --- 61.0 cms.

Position of Suspension	θ Radians	s_v	k_v	s_h	k_h	E Ergs per cubic cm
3.0 cms. Right	1.04	-0.51	-0.0220	-0.52	-0.0224	0.0459
2.0 " "	1.57	-0.53	-0.0228	-0.52	-0.0220	0.0570
0.0 " "	2.62	-0.32	-0.0138	-0.53	-0.0228	0.0736
2.0 " Left	3.26	-0.27	-0.0116	-0.32	-0.0138	0.0679
4.0 " "	2.72	-0.66	-0.0284	-0.27	-0.0116	0.0400
6.0 " "	1.40	-0.61	-0.0263	-0.66	-0.0284	0.0099

TABLE XLII

Frequency --- 45,000 cycles per sec.; Pendulum #3
- 1.515; -1.515; t - 0.339; K - 0.894
Series (1) See Table XXXVI & Figure 41 (1); ϕ - 0.0677
Voltage on Transmitter --- 2,700 volts; Distance from trans. 73 cms

Position of Suspension	θ Radians	s_v	k_v	s_r	k_r	E Ergs per cubic cm
6.0 cms. Right	1.23	-0.29	-0.0020	-0.27	-0.00186	0.00736
4.0 " "	1.76	-0.17	-0.00117	-0.26	-0.00166	0.00839
0.0 " "	2.37	-0.07	-0.00048	-0.09	-0.00062	0.00831
4.0 " Left	1.90	-0.15	-0.00104	-0.12	-0.00083	0.00604
6.0 " "	1.59	-0.16	-0.00110	-0.16	-0.00110	0.00471
Series (2) See Table XXVIII & Figure 41 (2). ϕ = 0.105 Voltage on Transmitter --- 4,000 volts; Distance from Transmitter ---130 cms.						
8.0 cms. Right	1.42	0.07	0.00075	0.07	0.00075	0.0075
4.0 " "	1.71	0.06	0.00064	0.07	0.00075	0.0094
0.2 " "	1.89	-0.05	-0.00054	0.03	0.00032	0.0119
4.0 " Left	1.65	-0.07	-0.00075	0.07	-0.00075	0.0109
8.0 " "	1.34	-0.07	-0.00075	-0.07	-0.00075	0.0085
Series (3) See Table XXXVIII & Figure 41 (3) ϕ = 0.105 Voltage on Transmitter --- 4,000 volts; Distance from Transmitter --- 166 cms						
6.0 cms. Right	0.78	-0.04	-0.00043	-0.033	-0.00035	0.00525
0.4 " "	0.96	-0.02	-0.00021	-0.030	-0.00032	0.00555
4.4 " Left	0.89	-0.01	-0.00011	-0.012	-0.00013	0.00545
8.8 " "	0.93	-0.028	-0.00030	-0.010	-0.00011	0.00524
13.0 " "	0.80	-0.04	-0.00043	-0.04-	-0.00043	0.00423

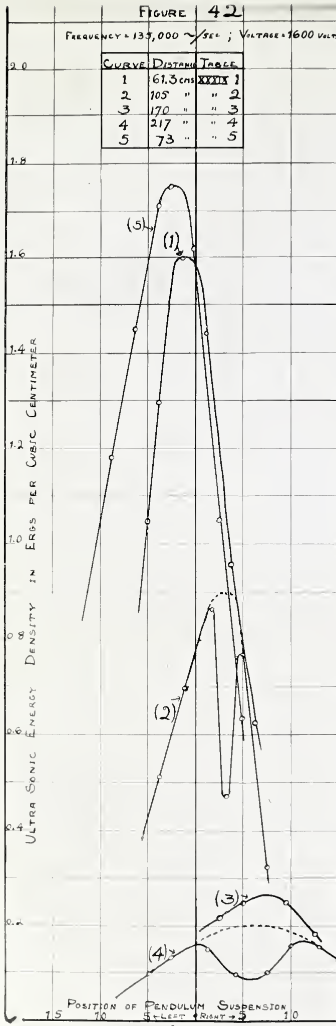


Figure 42
Frequency = 135,000 ~/sec ; Voltage = 1600 volts
Curve | Distance | Table
1 | 61.3 cms | XXXIX 1
2 | 105 " | " 2
3 | 170 " | " 3
4 | 217 " | " 4
5 | 73 " | " 5
(1) (2) (3) (4) (5)
2.0 1.8 1.6 1.4 1.2 1.0 0.8 0.6 0.4 0.2
Ultra Sonic Energy Density in Ergs per Cubic Centimeter
Position of Pendulum Suspension
15 10 5 ←Left 0 Right→ 5 10

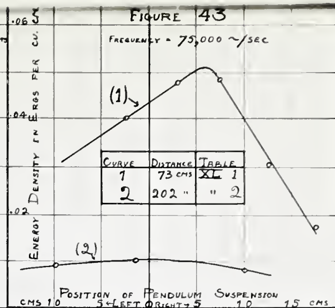


Figure 43
Frequency = 75,000 ~/sec
Curve | Distance | Table
1 | 73 cms | XL 1
2 | 202 " | " 2
(1) (2)
.06 .04 .02
Energy Density in Ergs per cu. cm
Position of Pendulum Suspension
cms 10 5 ←Left 0 Right→ 5 10 15 cms

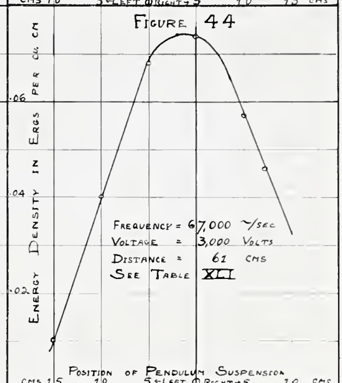


Figure 44
Frequency = 67,000 ~/sec
Voltage = 3,000 Volts
Distance = 61 cms
See Table XLI
.06 .04 .02
Energy Density in Ergs per cu. cm
Position of Pendulum Suspension
cms 15 10 5 ←Left 0 Right→ 5 10 cms

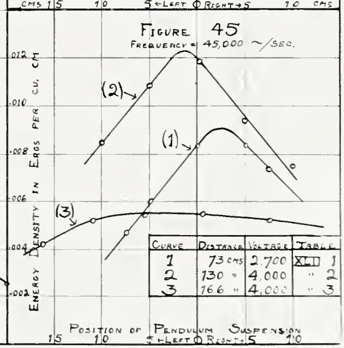


Figure 45
Frequency = 45,000 ~/sec.
Curve | Distance | Voltage | Table
1 | 73 cms | 2,700 | XLII 1
2 | 130 " | 4,000 | " 2
3 | 166 " | 4,000 | " 3
(1) (2) (3)
.012 .010 .008 .006 .004 .002
Energy Density in Ergs per cu. cm
Position of Pendulum Suspension
15 10 5 ←Left 0 Right→ 5 10

c. Energy Radiated by Transmitter at its Resonant Frequency

In figure 22, section 7, a horizontal section of the ultra-sonic beam at the resonant frequency of the transmitter is given. In section 11, figure 42, curve (5), the maximum intensity of this section is expressed in ergs per cubic centimetre. Onee the maximum intensity of the section shown in figure 22 has been determined the intensity at any point in the section may be obtained because as shown in section (4), and again in equation (6), the ultra-sonic intensity is approximately proportional to the pendulum deflections and therefore to the ordinates of figure 22.

If figure 22 is rotated about the ordinate passing through the peak of the section (0.0 cms) a solid of revolution will be formed which may be used as a representation of the central ultra-sonic beam. The volume of this solid of revolutions is a measure of the energy in the beam contained between two parallel sections one centimetre apart at the distance from the transmitter at which the considered section was taken (viz. 73 cms.).

If r = distance from central axis of figure 22.

and E = ultra-sonic intensity,

the volume of the solid of revolution would be

$$\int_0^{E_{max}} \pi r^2 dE$$

where E max. - intensity at the peak of the beam.

In the above integral r is a function of E, but the relation between r and E is very complex (see Verdet's equation). A simple way of obtaining an approximate value of the integral is by Simpson's method.

The values of r; πr^2; and E, determined from figure 22, are tabulated in Table XLIII. The values of πr^2 taken as ordinates are calculated over ten equal intervals of E, each interval being .1751 ergs per cc.

TABLE XLIII

Ordinate Number	E	r	πr^2
1	1.751	0	0
2	1.5759	1.25	4.90
3	1.4008	1.70	9.09
4	1.2257	2.10	13.82
5	1.0506	2.53	20.10
6	0.8755	2.95	27.4
7	0.7004	3.40	36.3
8	0.5253	3.85	46.5
9	0.3502	4.18	54.8
10	0.1751	5.10	81.8
11	0.0000	7.50	156.0

Ordinate 1 + Ordinate 11 = 156.0
(sum of odd ordinates) x 2 = 241.0
(" " even ") x 4 = 695.5
1092.5

By Simpson's rule the required integral = 1/3 x .1751 x 1092 = 63.9

Therefore the energy contained between two sections of the beam one centimetre apart at a distance of 73 centimetres from the transmitter amounts to about 64 ergs.

This energy was being propagated with the velocity of sound 1.5×10^5 cms. per second. Therefore the amount of energy radiated per second across this section of the main beam was $64 \times 1.5 \times 10^5$ ergs per second $= 9.6 \times 10^6$ ergs per sec. = .96 watts.* at a frequency of 135,000 cycles per second and a voltage of 1600 volts applied to the transmitter.

These figures apply to energy radiated from the first transmitter built, the one used throughout the work described in Parts I and II of this paper. Later

* Since these results were obtained transmitters with much greater radiating power have been constructed. See Part III

transmitters described in Part III had much greater radiating powers.

12. Electrical Resistance of Transmitter

The effective electrical resistance of the transmitter at a frequency of 135,000 cycles per second was determined. The electrical generating circuit (see figure 1) was set oscillating and the wave length, amperage and voltage were noted. The transmitter was then replaced by a variable condenser in series with a variable resistance. The condenser was adjusted until the wave-length of the circuit was the same as the wave-length of transmitter circuit, and the variable resistance was adjusted to give the same amperage as was obtained when the transmitter was in the circuit. Under these conditions the capacity of the variable condenser must equal the capacity of the transmitter, and, assuming that the resistance of the variable condenser was negligible, i.e. that the electrical losses were very small, the resistance in series with the condenser must equal the effective resistance of the transmitter. The following results were obtained:

Capacity of Transmitter = .0058 microfarads.
Resistance at frequency of 135,000 cycles per sec. = 155 ohms.

The current through the transmitter, when an oscillating voltage of 1600 volts at a frequency of 135,000 cycles per second was applied, was 0.6 amps. Therefore the power supplied to the transmitter was $155 \times (0.6)^2 = 56$ watts.

Power consumed by transmitter = 56 watts.
Power radiated by transmitter = 0.96 watts.
Efficiency of transmitter = 1.7%.

13. Effect of Distance from the Transmitter upon the Ultra-Sonic Energy

In figure 42, curves 1, 2, 3, 4, we have the absolute ultra-sonic intensity at different distances from the transmitter, all conditions other than the distance from the transmitter remaining ~~constnat~~ constant. With this data we can study the

law by which the ultra-sonic intensity falls off with distance. In Table XLIV the natural logarithm of the ultra-sonic intensities together with the logarithm of the corresponding distances from the transmitter are given, and in figure 46, the logarithm of the ultra-sonic intensity, $\log_e E$, is plotted against the logarithm of the distance, $\mathrm{Log}_e d$.

TABLE XLIV

Frequency --- 135,000 cycles per second.
Voltage --- 1,600 volts.

Distance from Transmitter	$\mathrm{Log}_e D$.	Ultra-sonic Intensity E. Ergs per cubic cm	Log_e E.
61.3 dms.	4.1158	1.60	0.4700
105. 0 cms.	4.6635	0.89	1.8835
170. 0 "	5.1358	0.27	2.6907
217. 0 "	5.3799	0.19	2.3393

If the radiation fell off exactly as the inverse square of the distance, the curve of figure 46 should be a straight line with a slope of (-2) but, figure 46 shows that the relation between log E and log D is not linear. For the larger values of log D the curve is approximately a straight line with a slope of about (-2) but nearer the transmitter the slope of the curve decreases considerably.

If we assume that the ultra-sonic energy is radiated in a beam of constant angular width, and that the vibrations are not damped while passing through the water, the energy should fall off inversely as the square of the distance from the transmitter. If, however, as will be the case, the viscosity of the water dampens the ultra-sonic vibrations in addition to the above decrease with the square of the distance, the energy will fall off exponentially with the distance from the transmitter and the expression for ultra-sonic intensity becomes:

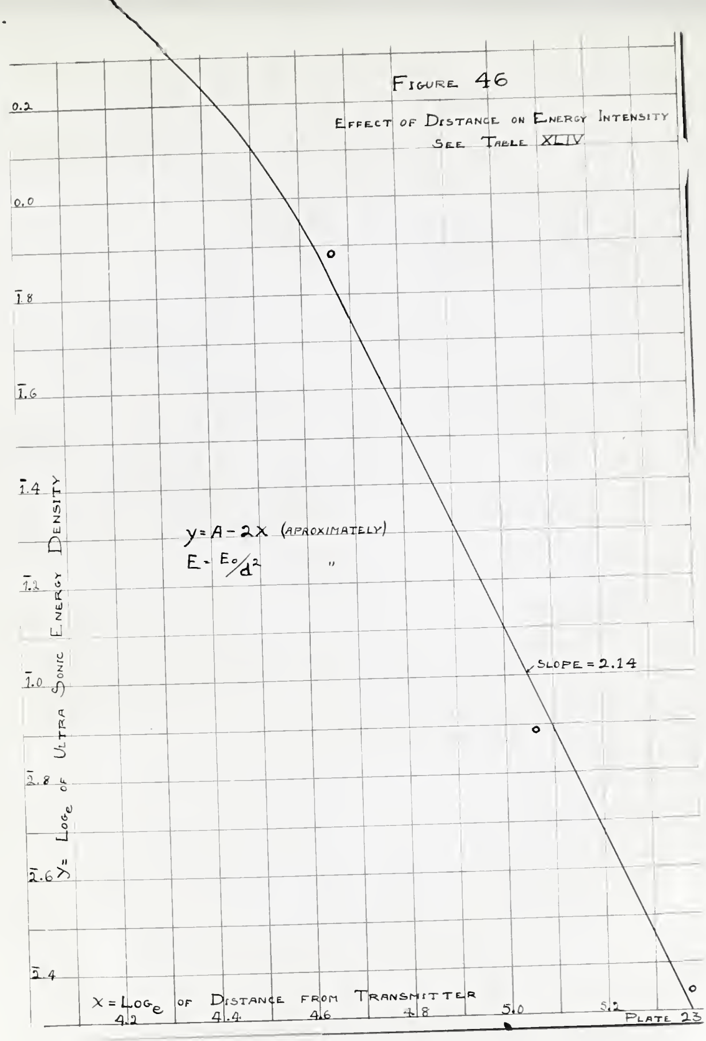


FIGURE 46
EFFECT OF DISTANCE ON ENERGY INTENSITY
SEE TABLE XLIV
0.2
0.0
1̄.8
1̄.6
1̄.4
1̄.2
1̄.0
2̄.8
2̄.6
2̄.4
y = Loge of Ultra Sonic Energy Density
y = A − 2X (APROXIMATELY)
E = Eo/d²
"
SLOPE = 2.14
X = Loge of Distance from Transmitter
4.2
4.4
4.6
4.8
5.0
5.2
PLATE 23

$$E = \frac{E_0}{d^2} e^{-md}$$

Where E_0 - initial ultra-sonic intensity

d - distance from transmitter.

m = damping constant

e = base of natural logarithms.

Taking $\log_e$ of the above expression we get

$\log_e E = A - 2 \log_e d - md$, where A is a constant.

If $\log_e E$ is plotted against $\log_e d$ and if $\log_e d = x$, $\log_e E = y$.

The curve obtained should correspond to the expression

$$y = A - 2x - me^x$$

In figure 46 at moderate distances from the transmitter the relation which occurs appears to be $y = A - 2x$.

Apparently, the damping constant m is very small so that, at the distances under consideration the factor me^x is not noticeable. Further work on this problem is anticipated and it is hoped that the effect of viscosity may be determined.

The fact that near the transmitter the ultra-sonic energy falls off less rapidly than the inverse square of the distance may be explained from a consideration of the results obtained in section 7 above. In section 7, it was found that near the transmitter the angular width of the ultra-sonic beam decreased as the distance from the transmitter increased. In view of this decreasing angular width of the ultra-sonic beam the ultra-sonic intensity should decrease less rapidly than the inverse square of the distance. At greater distances from the transmitter the angular width of the beam approaches a constant value and so the ultra-sonic intensity should fall off approximately inversely as the square of the distance.

14. Radiation Around Transmitter

In the above work only the ultra-sonic readiations from the face of the transmitter have been considered. It is interesting to see what energy is radiated from the sides and back of the instrument.

The torsion pendulum was set up 61 centimetres from the face of the transmitter and which was then rotated aroung a vertical axis. The results obtained are quoted in table XLV and plotted in polar coordinates in figure 47. Column θ in table XLVII designates the angular rotatio of the transmitter and column δ gives the ~~deflection~~ reading of the torsion pendulum. When the value of θ was zero the transmitter faced directly towards the pendulum, the peak of the central ultra-sonic beam striking the vertical pendulum vane. The experiment was carried out at a frequency of 135,000 cycles per second with a voltage of 1600 volts on the transmitter.

TABLE XLV

θ	δ = Pendulum Reading	Remarks
4°	7°	
5	8	
7	9.5	First Side beam.
9	6.0	
11	4.5	
13	0	
15	0	
30	0	
50	0	
73	0	Edge of transmitter plate towards pendulum.
90	0	

TABLE XLV (contin.)

θ	δ = Pendulum Reading	Remarks
120°	0	
135	0	
150	0	
180	0	Back of Transmitter to Pendulum
210	0	
225	0	
240	0	Edge of Transmitting Plate to Pendulum.
270	0	
287	0	
310	0	
330	0	
350	0	
335	0	
339	2	Second Side Beam.
341	3	
345	0	
351	3	First Side Beam
355	25	
356	37	
357	33	
357.5	35	
from 358 to 3.0	very large deflection. Max. deflection about 300°.	Main Central Beam.

Figure 47

Radiation Around Transmitter

See Table XLV

Angular Coordinate Represents Rotation of Transmitter
Vector Coordinate Represents Pendulum Deflections

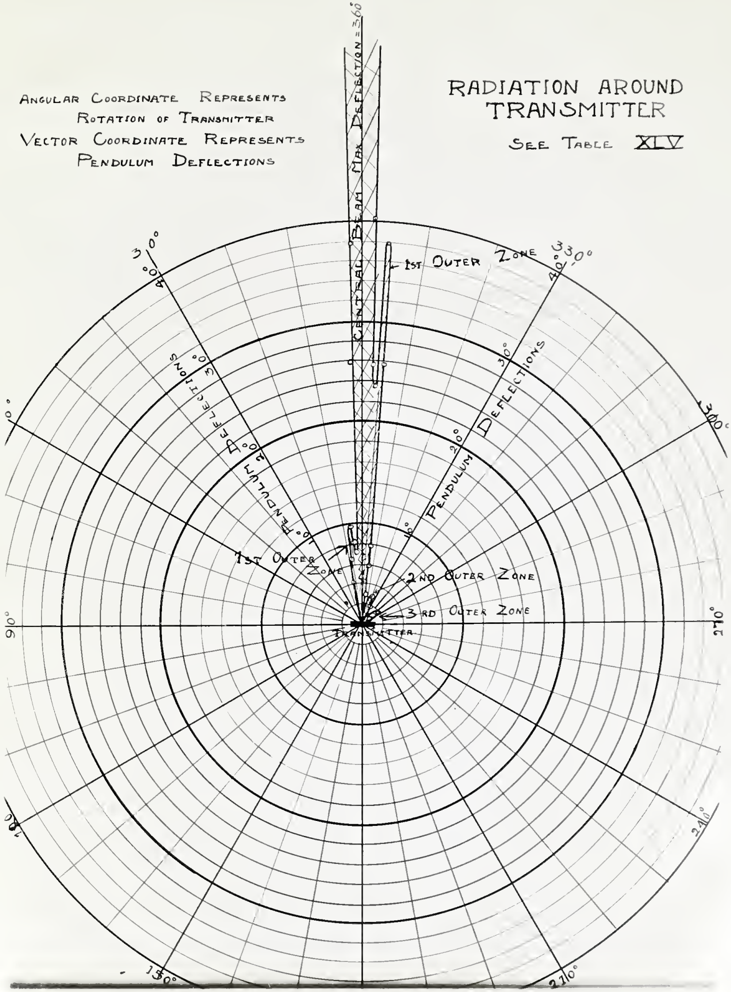


Figure 47 shows that no energy is radiated from the back or sides of the transmitter. Aside from the energy radiated in the central ultra-sonic beam, the only energy radiated by the transmitter is in the side beams investigated in section 7 (d).. The energy radiated in these side beams is extremely small and is negligible in comparison with that of the main beam. Referring to section 4 it is seen that the ultra-sonic energy is generated by the piezo-electric displacements in a quartz mosaic. These piezo-electric displacements should generate ultra-sonic oscillations at the back face of the plate as well as at the front. The fact that no energy is detected from the back of the instrument probably means that all the energy radiated by the back face of the quartz is absorbed in pitch and wood which forms the protective covering in the back of the instrument.

PART THREE

PART 111

Preliminary study of Ultra Sonic Qscillators.

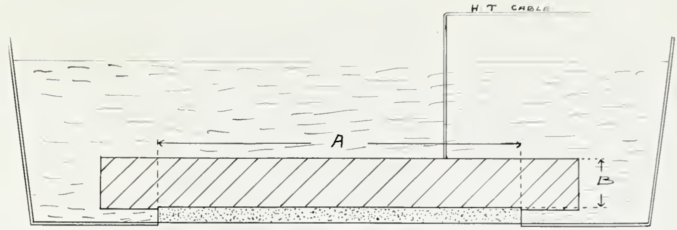
In Parts 1 and 11 we have been considering the energy of the ultra sonic beam without any serious consideration of the source of that energy. We now pass on to a dicsussion of the behaviour of various types of ultra sonic transmitters.. A number of instruments have been built and studied and some attempt has been made to explain the behaviour of these but it must be considered that the present investigation is not complete. Further experiments are being cattied on at the time of writing.

The principle underlying the operation of an oscillator or transmitter has been described in section 2 of Part 1. The various oscillators considered here may be divided into four distinct types, depending upon the arrangement of the metal electrode and the quattz mosaic in the instrument. These types have been designated A,B,C and D.

Type A has already been described in section 2 of Part 1. It consisted of a quartz plate separated from the water in the tank by a thin sheet of mica which kept the instrument water-tight. The quartz was backed by a circular steel plate which served as one electrode of the electrostatic field. The water in the tank served as the second electrode.

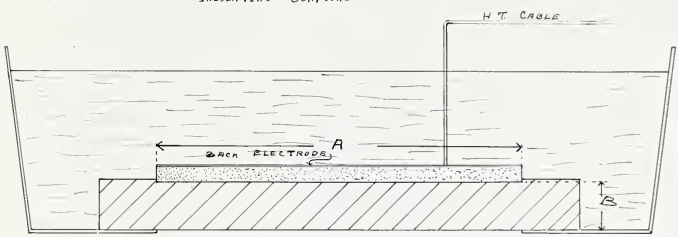
Type B is the reverse of type A and in these instruments the steel plate was next the water and in contact with it. The quartz was placed immediately behind the steel and was in turn backed by a thin sheet of metal foil which served as the second electrode.

Figure 48

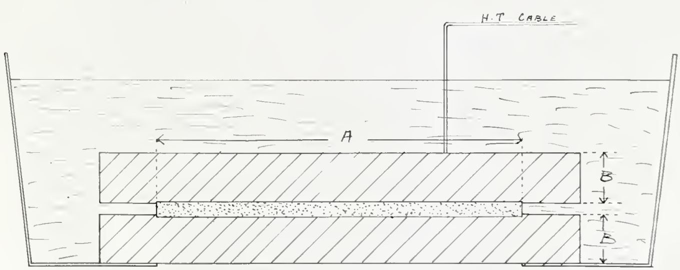


(A) Quartz Plate to Water.

///// Denotes Steel
" Quartz
" Insulating Compound



(B) Steel Plate to Water



(C) Double Steel Plates.

Plate 26

Type C was a combination of types A and B and in this type of instrument, called the double plate instrument, the quartz was placed between two steel plates of equal thickness. The front plate was in contact with the water and the back plate was connected to the high potential terminal of the electrical oscillation circuit.

Type D was rather different from any of the above. All heavy metal plates were dispensed with and the requisite thickness was obtained by piling up layers of quartz. Each layer was separated from the adjacent ones by thin copper sheets .046 cms. thick. These copper sheets served as the electrodes in the instrument. When building this type of transmitter care had to be taken to have the direction of the distortion, for any given electric field, the same in all layers of quartz, otherwise the distortion in one layer might neutralize that in the next. Each section of the quartz plates had one face marked positive. This indicated that on testing the quartz the marked surface had given a positive charge when subjected to pressure. When building the oscillator the positive faces of two adjacent layers of quartz were placed in contact with the same metal electrode.

Sketches of type A, B and C are shown in figure 48.

15. Single Steel Plate Oscillator with Quartz to Water.

Type A.

(a) Theoretical Consideration and Experimental Procedure.

In section 6 of Part 1 it has been shown that a transmitter must have a definite frequency of vibration, the natural frequency of longitudinal vibration, at which the

ultra sonic radiation from the instrument has a maximum intensity. In order to be in a position to design a satisfactory ultra sonic transmitter it is necessary to know the factors which control the ~~factors~~ frequency of maximum energy emission.

The frequency of maximum energy emission must be dependent upon the thickness of the steel back plate in the instrument. When the frequency of the longitudinal vibrations in the quartz coincides with the natural period of free longitudinal vibration of the steel plate the steel will be set into a strong resonant vibration. Under these conditions the ultra sonic energy would be generated not only by the piezo-electric vibrations of the quartz but also by the resonant vibrations in the steel and the energy radiated should therefore be a maximum. The natural period of longitudinal vibration of a rod free at both ends occurs when the length of the rod is equal to any integral number of half-wave-lengths. We should therefore, on simple theory, expect to obtain resonance in the ultra sonic transmitter when the thickness of the back plate was equal to an integral number of half-wave-lengths of the ultra sonic wave in steel, corrected, possibly for other accompanying phenomena.

To test this relation a large number of steel plates of varying thickness were prepared. Instruments were then built with these various plates as back plates. The transmitter under consideration was set oscillating and the torsion pendulum was placed at the point of maximum intensity of the ultra sonic beam, at a distance beyond the region of interference zones discussed in section 7 (c).

The frequency of the electric oscillation applied to the transmitter was then varied by means of the variometer shown in Fig. 2. The pendulum readings for the various frequencies were noted. These pendulum readings were then reduced to energy densities by the method discussed in section 11 of Part II. and from the energy densities the amplitudes of vibration were determined. Curves were plotted in which the amplitude of vibration of the wave in water at the position of the pendulum were represented as ordinates and the corresponding frequencies as abscissae. These curves are called the "characteristic curves of the transmitter."

The first experimental type of instrument gave results which are not now regarded as reliable. When these instruments were made care had to be taken to remove all traces of air from the seams in the quartz mosaic and from the quartz-steel surface. This was done by spreading vaseline on the steel plate, in rubbing the quartz on the vaselined surface and forcing out as much of the vaseline as possible. When the instrument is built in this way the junction between steel and quartz is unsatisfactory though the instrument is good enough to demonstrate all the phenomena involved.

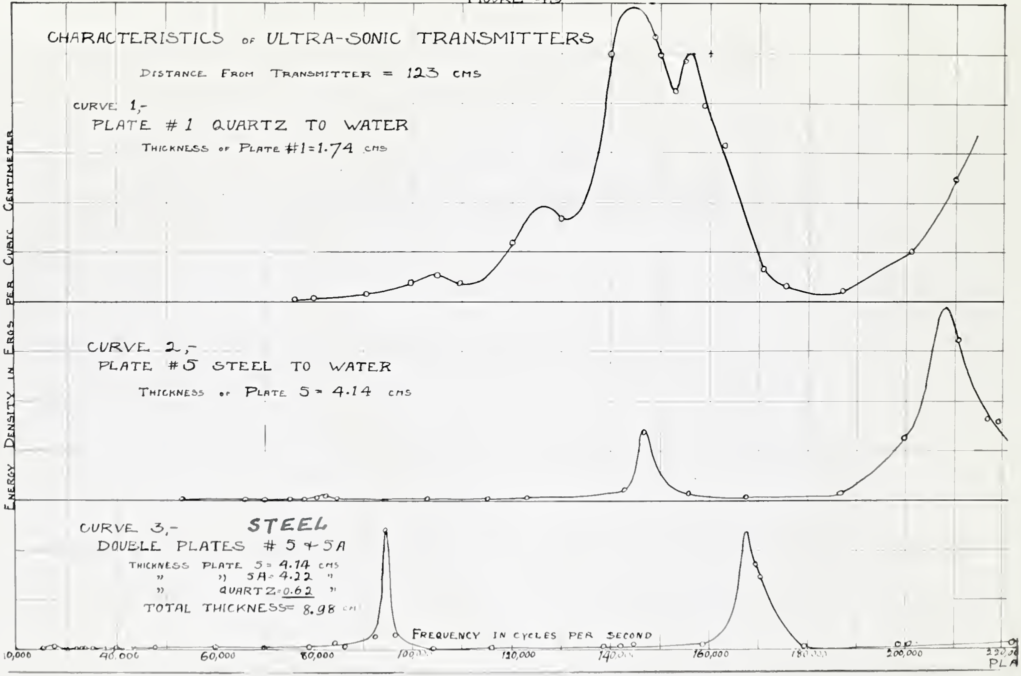
At the time we performed the first series of experiments we had, as a source of electric supply, only the alternating line voltage, and it was difficult to adjust the oscillating line voltage and frequency applied to the transmitter. We found that with the above construction of instrument and the difficulties of voltage and frequency control, we did not get such clear and consistent results as later. The characteristic curve of the instrument showed, in addition to one main frequency of maximum energy emission, a number of irregular sec-

ondary maxima. In later instruments the vaseline in the quartz -steel interface was replaced by a mixture of wax and resin which constitutes a much better cement. The hot mixture was poured over the steel, which had also been heated, and the quartz was pressed down as before. On cooling the wax and resin solidified and the quartz was solidly cemented to the steel. This method of construction yielded a much more reliable type of instrument. Also by the time we were ready for the later series of experiments we had secured a 2,000 volt, direct current supply for the plates of the oscillating valves and most of the difficulties in the control of voltage and frequency were over.

In the later series of experiments three transmitters were built with back plates of the same quality of steel but different thickness. The characteristic curves of these were taken and the results obtained were tabulated on the tables XLV11, XLV111 and XL1X. In Fig. 49 the characteristic curve of one of the transmitters, based on energy densities instead of amplitudes of vibration , is given, together with typical curves for instruments of types B and C.

These curves show the very sharp resonance conditions which are met with. The peaks for the double-plate instruments of type C are extremely narrow but even in the case of transmitters of type A frequency fluctuations of only 3,000 or 4,000 cycles about 2% or 3% of the resonant frequency, may produce energy fluctuation as large as 25% or 30%. It is therefore a matter of considerable importance to determine the position of the resonant peaks in the characteristic curve of an instrument and also the factor which controls the position

FIGURE 49



CHARACTERISTICS of ULTRA-SONIC TRANSMITTERS
DISTANCE FROM TRANSMITTER = 123 CMS
CURVE 1,-
PLATE #1 QUARTZ TO WATER
THICKNESS of PLATE #1=1.74 CMS
CURVE 2,-
PLATE #5 STEEL TO WATER
THICKNESS of PLATE 5 = 4.14 CMS
CURVE 3,- STEEL
DOUBLE PLATES #5 + 5A
THICKNESS PLATE 5 = 4.14 CMS
" " 5A = 4.22 "
" QUARTZ = 0.62 "
TOTAL THICKNESS = 8.98 CM
ENERGY DENSITY IN ERGS PER CUBIC CENTIMETER
FREQUENCY IN CYCLES PER SECOND
10,000
40,000
60,000
80,000
120,000
160,000
200,000

of these peaks.

Although the energy density of the ultra sonic vibration is the factor which is of practical importance it is necessary to contrast the amplitudes of vibration, rather than the energy densities, in a comparison of the characteristic curves of the instrument. Referring to the familiar expression for the energy density of a sound wave, viz:

$$E_o = 2\pi^2 a^2 n^2 p \qquad \text{Equation (1)}$$

where E_o = energy density in ergs per cu.cm.

a = amplitude of vibration in cms.

n = frequency of vibration.

p = density of the medium.

it is evident that, aside altogether from resonance conditions, the energy density will increase as the square of the vibration frequency. On the other hand fluctuations in the amplitude of vibration, with varying frequencies, can be due only to a resonant effect.

In Fig. 50A the characterisitc curves of the transmitters under consideration are given. These curves are based on amplitude of vibration. It will be noticed that the amplitude curves are not as sharp as the energy density ones on account of the fact that the amplitude is proportional to the square root of the energy density. Curve 1 is the characteristic of a transmitter with a steel back-plate 1.74 centimetres thick; curve 2 for a plate 2.08 cms thick and curve 3 for a plate 4.14 centimetres thick. In all three cases the same quartz plate was used; its thickness was 0.62 centimetres and its diameter 15.2 centimetres.

The ultra sonic energy densities were obtained from the pendulum reading by equation 6 of section 11(a) Part 11.

In the following series of experiments a lead pendulum with vanes 0.258 centimetres thick and 1 centimetre in diameter was used. It follows from the results quoted in section 9 of Part 11 that the effect of the thickness of the vane of this pendulum is constant over a range of frequencies from 100,000 cycles per second to 300,000 cycles per second. The value of the constant K, discussed in section 9, was 0.99 over this frequency range. For frequencies below 100,000 cycles per second an appropriate correction for the thickness of the pendulum vane must be made. The corrected values of K, where necessary, are given in the tables below.

In the tables the frequencies of the electric oscillations are quoted in the first column; the corresponding pendulum reading measured in radians in the second column; the energy densities in ergs per cubic centimetre in the third and the amplitudes of vibration in the fourth.

The wave ~~point~~ front in the ultra sonic beam is not plane and there is, of course, no constant value of phase at different points in any section of the beam. Yet over the small area covered by the pendulum vane the wave can be considered as plane without significant error, and in the calculation of amplitudes from energy density we need not be concerned with the vibration phase. Hence the amplitude in the water can be calculated from the relation expressed in equation 1 of this section.

A factor, which is of considerable importance in the consideration of the characteristic curves, is the concentration of the ultra sonic energy as the frequency of vibration increases. It has been shown in section 7 of

Part 1 that the width of the ultra sonic beam decreases as the frequency increases. This concentration of the ultra sonic energy must result in an increase in the amplitude of vibration at the centre of the beam. In many of the characteristic curves it will be noticed that the amplitude of vibration in the second and higher harmonics of the fundamental resonant frequency is usually as great or even greater than the amplitude of vibration at the fundamental. This effect is, no doubt, due to the concentration of the ultra sonic energy at the higher frequencies.

The following factors are constant throughout tables XLVII, XLVIII and XLIX:-

1. The voltage impressed on the transmitter was 3,000 volts.

(Note:- This voltage could not be kept absolutely constant throughout a long series of experiments but, by utilizing the voltage square law established in section 10 all readings have been corrected to correspond to a voltage of 3,000 volts)

2. The energy density from equation 6 of section 11 Part II is

$$E_o = .227 \text{ Ergs per cubic centimetre}$$

for frequencies greater then 100,000 cycles per second. For frequencies lower than 100,000 cycles a correction has to be made for the lower values of K.

3. The amplitude of vibration, from equation 1 of this section is

$$a = .226 \sqrt{\frac{E_o}{n^2}} \text{ centimetres.}$$

TABLE XLVII.

THICKNESS OF STEEL BACK PLATE 1.74 cms.

Frequency. Cycles per second.	Pendulum Reading in Radians.	Energy Density Ergs per cu.cm.	Amplitude of Vibration.	Remarks.
28,300	.054	.0187	1.09×10^{-6} cms	Harmonic of Freq. 14300 K = .65
36,600	0	0	0	Harm. of Fr 214,000
38,800	.250	.063	1.50	K = .89
40,800	.010	.0025	.28	K = .90
42,400	.010	.0025	.27	
44,400	0	0	0	
45,200	.034	.0083	.45	K = .915
46,800	.098	.025	.76	K = .92
47,700	.104	.025	.75	Harm. of Fre. 210,000 K = .93
48,000	0	0	0	
52,700	.158	.0377	.82	
53,400	.158	.0377	.62	K = .94
56,500	.223	.0533	.92	
61,000	.540	.126	1.32	K = .96
71,500	1.02	.235	1.53	
75,000	1.24	.286	1.61	K = .975
75,000	1.37	.320	1.70	
81,100	1.22	.280	1.47	K = .98
Moved out to 132 cms. from transmitter.				
76,000	.158	.0365	.57	K = .975
80,000	.400	.092	.86	K = .98
90,400	.839	.190	1.09	K = .99
99,300	1.84	.42	1.47	K = .99
105,000	2.39	.54	1.58	
109,500	1.86	.43	1.35	
120,000	5.40	1.22	2.08	
130,000	7.51	1.70	2.27	
140,000	21.2	4.81	3.54	Distinctly two peaks.
149,000	23.6	5.35	3.51	
153,000	19.0	4.31	3.07	
155,000	21.5	4.87	3.21	
159,000	17.5	3.97	2.84	
163,000	14.0	3.18	2.47	
171,000	3.02	.69	1.09	
175,600	1.57	.356	.77	

TABLE XLV11. (Cont'd.)

Frequency. Cycles per second.	Pendulum Reading in Radians.	Energy Density in Ergs per Cu.Cm.	Amplitude of Vibration.	Remarks.
188,800	1.25	.274	.63 $\times 10^{-6}$ cms	K = .99
201,000	4.72	1.07	1.16	
210,000	11.48	2.60	1.73	
223,000	14.20	3.20	1.82	
236,000	37.00	8.40	2.78	

TABLE XLV111

Thickness of steel back-plate 2.08 cms.

15,800	.006	.00225	.68 $\times 10^{-6}$ cms	K = .60
33,000	.011	.00278	.36	K = .85
35.200	.019	.00491	.45	" " .87
35,800	.025	.00642	.49	" .875
36,500	.014	.00358	.37	" .88
38,400	.031	.00786	.54	" .885
38,600	.132	.0355	1.08	" "
39,200	.115	.0296	.80	" .89
40,500	.020	.0050	.39	" .90
54,600	.131	.0330	.75	" .94
58,200	.195	.0459	.83	" .955
66,100	.233	.0539	.80	" .970
68,200	.311	.0721.	.75	" "
70,100	.327	.0754	.89	" .975
70,900	.275	.0634	.80	" "
72,500	.494	.1145	1.05	" .975
74,400	.558	.1298	1.09	" "
77,700	.589	.136	1.08	" "
83,300	1.115	.264	1.40	" .980
91,000	2.62	.600	1.93	" .990
96,800	3.20	.734	2.00	" "
	Transmitter moved to 123.5 cms.			
80,000	.269	.059	.69	" .98
84,000	.839	.192	1.17	" "
89,000	1.20	.275	1.34	" .99
95,000	.96	.218	1.12	" "
105,000	3.54	.804	1.93	" "
118,000	8.38	1.90	2.64	" "
125,000	7.01	1.59	2.28	" "
143,000	2.16	.49	1.11	" "
150,000	.87	.20	.67	" "
170,000	3.37	.77	1.17	" "
192,500	12.00	2.73	1.94	" "
214,000	45.10	10.20	3.48	" "

TABLE XLV111 (Cont'd)

Frequency. Cycles per second.	Pendulum Reading in Radians	Energy Density in Ergs per cu.cm.	Amplitude of Vibration.	Remarks.
219,000	54.00	12.25	3.61 $\times 10^{-6}$ cms.	K = .99
219,000	61.80	14.05	3.88	
224,000	45.30	10.25	3.21	

TABLE XL1X

Thickness of steel back-plate 4.14 cms.

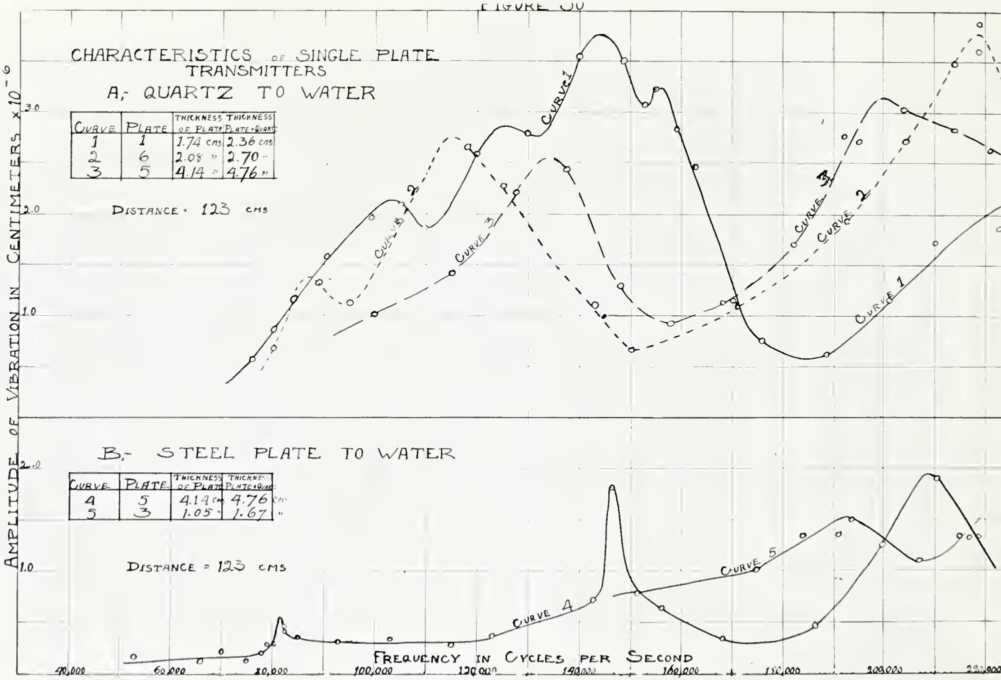
Frequency. Cycles per second.	Pendulum Reading in Radians	Energy Density in Ergs per cu.cm.	Amplitude of Vibration.	Remarks.
100.000	.874	.198	1.005 $\times 10^{-6}$ cms	
115,000	2.24	.510	1.401	
127,500	6.95	1.572	2.22	
137,500	8.93	2.20	2.44	
148,000	3.20	.726	1.30	
158,000	2.15	.484	.994	
168,000	3.13	.710	1.133	
182,000	8.47	1.920	1.720	
192,200	24.3	5.52	2.76	
195,000	24.3	5.52	2.72	
204,000	32.9	7.47	3.03	
204,000	26.9	6.11	2.74	
214,000	31.9	7.24	2.84	
221,000	30.0	6.81	2.67	

(b) Discussion of Results.

Referring to Fig. 50 A we see immediately that the thick plates do not have as large a resonant amplitude as do the thinner ones. The peak value of curve 3, for which the back-plate was 4.14 cms. thick amounts to only about 70% of the peak value of curve 1 in which the thickness of the plate was 1.74 cms. Curve 2, for which the back-plate was 2.08 cms thick gives peak values intermediate between those of curves 1 and 3. The reason no doubt is that there is appreciable energy loss due to viscosity in the metal plates. This loss would, of course, not be so great in a thin plate as in a thicker one.

Considering the position on the characteristic curves of

FIGURE 30



CHARACTERISTICS OF SINGLE PLATE TRANSMITTERS
A,- QUARTZ TO WATER
CURVE | PLATE | THICKNESS OF PLATE | THICKNESS PLATE+QUARTZ
1 | 1 | 1.74 cms | 2.36 cms
2 | 6 | 2.08 " | 2.70 "
3 | 5 | 4.14 " | 4.76 "
DISTANCE = 123 CMS
CURVE 1
CURVE 2
CURVE 3
B,- STEEL PLATE TO WATER
CURVE | PLATE | THICKNESS OF PLATE | THICKNESS PLATE+QUARTZ
4 | 5 | 4.14 cms | 4.76 cms
5 | 3 | 1.05 " | 1.67 "
DISTANCE = 123 CMS
CURVE 4
CURVE 5
AMPLITUDE OF VIBRATION IN CENTIMETERS x10^-6
3.0
2.0
1.0
2.0
1.0
FREQUENCY IN CYCLES PER SECOND
40,000
60,000
80,000
100,000
120,000
140,000
160,000
180,000
200,000
220,000

the resonant frequencies, the results indicate that resonance occurs when the thickness of the back-plate is approximately equal to an integral number of half-wave lengths in the metal plate. Referring to curve 1 we find the resonant peak at a frequency of 145,000 cycles per second. Assuming for the present discussion that the velocity of sound in steel is 5.1×10^5 centimetres per * second this resonant frequency corresponds to a wave-length in steel of 3.52 centimetres.

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* This value is the one often quoted for low frequency longitudinal elastic vibrations, but it is calculated from a value of Young's Modulus determined statically. Experiments recently performed in this laboratory indicate that this value is not far from the correct velocity of longitudinal waves in steel rods for frequencies up to 60,000 vibrations per second.

The back-plate in the instrument under consideration was 1.74 centimetres thick and therefore resonance occurs when the thickness of the back-plate is 0.98 half wave-lengths. This is very nearly one-half wave-length.

In curve 2 we have resonance at frequencies of 115,000 cycles per second and 219,000 cycles per second. These frequencies correspond to wave-lengths in steel of 4.43 cms. and 2.23 cms. The steel plate of this instrument was 2.08 cms. thick and therefore resonance occured when the thickness of the plate was 0.94 half wave-lengths, approximately one half wave-length, and 1.82 half wave-lengths or approximately two half wave-lengths.

The transmitter referred to in curve 3 had a back-plate 4.14 centimetres thick. Resonance occured at frequencies of 134,000

cycles per second and 200,000 cycles per second. The corresponding wave-lengths in steel are 3.78 and 2.55 centimetres. We have resonance therefore when the thickness of the back-plate is 2.18 half wave-lengths, approximately ~~3~~ 2 half wave-lengths and 3.14 half wave-lengths or approximately 3 half wave-lengths. In other words this plate was vibrating not to the note of its natural fundamental vibration but to sequent higher tones. The fundamental has, however, been located for thinner plates in curves 2 and 3.

It must be remarked that in the experiments just described it is rather difficult to locate the peaks of energy density with accuracy on account of their very sudden rise and fall as the resonant peak is passed. Also it may be necessary to have the wave-meter used re-standardised* to make more certain of the exact frequencies; and in any case we do not know the correct velocity of such high frequency ~~ies~~ waves in steel. But the results discussed here indicate clearly, as might be expected theoretically, that resonance occurs when the thickness of the steel plate is approximately an integral number of half wave-lengths. This relation is however at present only an approximation. The discrepancies which occur in curves 1 and2 are greater than the limits of experimental error might allow, and in curve 3, although a maximum resonant peak occurs when the thickness of the steel plate is one half wave-length within the limits of error, secondary resonant peaks on both sides of the main one appear. This is not surprising. The half wave-length ~~theory~~ hypothesis takes no account of the presence of a quartz plate of finite thickness comparable with that of the steel back-plate. There can be no doubt that the quartz

*Restandardization has shown that the frequencies quoted in this paper are 3% higher than they should be

plate must have an effect on the resonance condition of the instrument as a whole. If we assume that the piezo-electric displacements in the quartz are emitting wave trains of the form $Ae^{i(pt \pm \frac{x}{v})}$ forward toward the water and $Be^{i(pt \mp \frac{x}{v_1})}$ backward into the steel where x denotes distance from the quartz

v and v_1 denote the velocities of sound respectively in the medium considered

t denotes time

p is a periodicity factor

i equals $\sqrt{-1}$

and A and B are amplitude terms

then these wave trains will be reflected backward and forward between the water-quartz surface in front, the steel-insulating compound surface at the back of the instrument and reflections also will occur at the quartz-steel interface. A mathematical analysis of the problem, based on the above suggestion has been commenced but has not as yet been taken far enough to include in this present report. Indications have been obtained that the multiple peaks exhibited in curve 1 are to be expected, but as yet the relation between the frequency corresponding to these peaks, the thickness of the steel plate, the thickness of the quartz plate and the frequency of vibration has not been established. The investigation is being continued both experimentally and mathematically and further results will be reported at a later date.

16. Single metal plates, metal to water.

a. Two type B transmitters have been built. In these instruments the steel plate was placed in contact with the water; behind the steel the quartz plate was cemented with the

insulating mixture and behind this again there was a thin metal foil serving as the high tension electrode. In one of these instruments the steel plate of thickness 4.14 cms, which was considered in section 15 has again been utilized, and it is interesting to compare the characteristic curve of this instrument with that of the instrument, containing the same plate, of type A. The second transmitter had a steel plate of thickness 1.05 cms.

The data for these instruments was obtained in the same way as that for the instruments of type A. Table I contains the data for the instruments with the 4.14 cm. plate, and table II that for the instruments with the 1.08 plate. The characteristic curves of these instruments are plotted in Fig. 50B. Curve 4 refers to the instrument with the 4.14 cm. plate and curve 5 to that with the 1.08 cm. plate.

The following data holds throughout tables I and II :-

1. The voltage applied to the transmitter was 3,000 volts.
2. From equation 6 of section 11 (a) Part II the energy density is given by the relation

$$E_o = .227\,\theta \text{ ergs per cubic centimetre.}$$

$$\theta = \text{pendulum reading.}$$

(Note; for frequencies below 100,000 cycles per second the reflection constant K of equation 6 section 11 must be corrected for the thickness of the pendulum vane. See section ~~10~~ 15 (a).

3. From equation 1 of section 15 (a) the amplitude of vibration is given by

$$a = .226\sqrt{\frac{E_o}{n^2}} \text{ centimetres.}$$

TABLE L. Thickness of steel plate 4.14 cms.

Frequency. Cycles per second.	Pendulum Reading in Radians.	Energy Density in Ergs per Cu. Cm.	Amplitude of Vibration.	Remarks.
53,000	.0069	.0016	.173 x 10^{-6} cms	K = .95
66,000	.0074	.0017	.14 cms	K = .970
70,000	.0204	.0047	.22	" .974
75,000	.0115	.0027	.16	" .978
78,300	.0204	.0047	.20	" .98
79,000	.0420	.0096	.28	" "
80,500	.0475	.0109	.29	" "
82,500	.1195	.0274	.45	" "
82,500	.1075	.0248	.43	" "
85,000	.0752	.0170	.35	" .984
93,000	.0695	.0158	.31	" .988
103,000	.1010	.0232	.33	" .99
115,000	.0875	.0199	.28	" "
123,000	.174	.0390	.37	" "
143,000	.978	.222	.73	" "
146,300	6.08	1.38	1.82	" "
156,000	.852	.194	.65	" "
168,500	.310	.0710	.36	" "
187,500	.675	.156	.48	" "
200,000	5.62	1.28	1.28	" "
211,000	14.05	3.22	1.91	" "
217,000	7.38	1168	1.35	" "
219,000	7.10	1.61	1.35	" "

TABLE L1 Thickness of steel plate 1.08 cms.

151,500	1.26	.286	.80 x 10^{-6} cms	
171,500	2.70	.613	1.04	
184,000	5.51	1.250	1.37	
191,000	5.90	1.340	1.37	
193,500	7.38	1.673	1.51	
207,000	4.55	1.031	1.11	
214,000	6.75	1.531	1.36	

(b) Discussion of Results.

Comparing the characteristic curves of the transmitters of type A and B which were built with the same steel plate (see curve 3 and 4 of Fig. 50) we see that the resonant amplitude of type B is only about 70% of the corresponding resonant amplitude of type A. Since the energy radiated is proportional to the square of the amplitude of vibration it is evident that an instrument of type B radiates only half as much energy as a similar instrument of type A

This is no doubt due to the fact that in type B all the energy before it can reach the water must pass through the steel plate and be subjected to viscosity damping, whereas in instruments of type A a considerable amount of energy may be radiated directly from the quartz without having to pass through the steel.

A second interesting point which may be brought out by a comparison of curves 3 and 4 is the increase in the resonant frequency of the instrument of type B as compared with the corresponding resonant frequency of the type A. The resonant frequencies of the type B instrument appear to be about 12,000 cycles per second or 9% higher than the corresponding resonant frequencies of the type A instrument. This cannot be explained on the simple half wave-length hypothesis; but, as has already been pointed out in section 16 there are some divergencies from simple theory to be expected, and any explanation of these discrepancies will have to await a more or less rigid mathematical analysis of the problem.

One factor which would very probably affect the resonant frequency of ultra sonic transmitters is the lateral vibrations which must be occuring in the instrument. When a mass of metal is set into longitudinal vibrations, because of the effect of Poisson's ratio, lateral vibrations must also occur. Lord Rayleigh* has shown that when a rod is set into longitudinal vibrations the energy expended in longitudinal vibration is to the total energy expended in the rod, in both longitudinal and lateral vibrations as

$$\frac{1}{1 + \frac{\pi^2 i^2 p^2 r^2}{4 l^2}}$$

*(Lord Rayleigh; Theory of Sound. Vol. 1 Pages 251-253)

i is an integer denoting the mode of vibration, i.e: whether the rod is vibrating at its fundamental resonant frequency or at a higher harmonic.

p is Poisson's ratio for the material under consideration.

r is the radius of the rod.

l is the length of the rod..

A consideration of the above expression shows that considerable energy must be radiated from the sides of an ultra sonic transmitter. Considering the instrument with a steel plate 4.14 centimetres thick operating at it's fundamental resonant frequency we get

$$i = 1$$
$$p = 0.31$$
$$r = 9.53 \text{ cms.}$$
$$l = 4.14 \text{ cms.}$$

and Rayleigh's Ratio becomes $\dfrac{1}{2.26}$

Therefore the energy radiated per unit area from the face of the transmitter is tothe energy radiated per unit area from the side of the instrument as 1 is to 1.26. The area of the face of the plate under consideration is 285 square centimetres and the area of the side of the plate is 248 square centimetres. The ratio between the total energy radiated from the face of the transmitter and the total energy radiated from the side is therefore

$$\frac{1 \times 285}{1.26 \times 248} = \frac{285}{312}$$

and we see that the radiation from the face of the instrument amounts to only about 50% of the total energy radiation. The effect that the energy expended in lateral vibration has on the resonant frequency of a transmitter has not as yet been determined. Without considering these lateral vibrations it has been

found that at the fundamental note resonance occurred when the thickness of the plate was approximately half a wave-length. It would therefore seem probable that, at least for the fundamental mode of vibration, the correction for lateral vibration would not be very large.

17. Double Plate Transmitter

(a) Experimental Data

In section 15 and 16 we have considered ultra-sonic transmitters which have single steel plates behind and in front of the quartz plate respectively. We shall now consider an instrument consisting of two similar metal plates between which is the quartz. The metal plate served as the electrodes for the electro-static field across the quartz.

From a consideration of the piezo-electric displacements in the quartz we see that there must be, at the centre of the quartz, a place of maximum pressure fluctuation and zero displacement. At the outer surfaces of the metal plates the opposite conditions must occur, the pressure variations being minimum and the displacements a maximum. The theory of longtitudinal vibrations in bars free at both ends, shows that resonance should occur when the total thickness of the two plates is equal to an integral number of half wave-lengths. The added restriction, that an antinode of pressure or node of displacement occurs at the centre of the instrument limits the possible resonant conditions, and under this restriction resonance can occur only when the thickness of the instrument is an odd number of half wave-lengths.

a number of instruments of this type have been built and their characteristic curves taken in exactly the same way as described in section 15. Two instruments with steel plates have been investigated, and one with lead plates. In another instrument a non-conducting substance, viz. marble, was used for the plates, the electrostatic field being impressed on the quartz by thin lead foils cemented between the quartz and the marble. The results obtained have been tabulated in tables LII to LV and plotted in figure 51. Curve 1 of figure 51 refers to a transmitter with steel plates. The front plate of this instrument was 4.14 cms. thick and the back plate 4.22 cms. Curve 2 is the characteristic of the second steel plate transmitter. In this instrument the thickness of the front plate

was 1.74 cms. and that of the back plate 1.69 cms. The characteristic of the marble transmitter is given in curve 3, the thickness of the front and back plates being respectively 1.69 and 1.74 cms. Curve 4 gives the characteristic of the lead plates transmitter, in which the front plate was 0.57 cms thick, and the back plate 0.60 cms.

In table LII the pendulum deflections for frequencies greater than 80,000 cycles per second were obtained at a distance from the transmitter of 122.5 cms. Owing to the fact that the energy density is proportional to the square of the frequency the pendulum deflection for frequencies lower than 80,000 cycles per second were too small to be detected at this distance and the pendulum had to be moved to a distance of 32 cms. from the transmitter. The pendulum could not be left in this position at higher frequencies because of the presence of the interference zones discussed in section 7 (c) of Part I. Figure 16 shows that at a frequency of 71,000 cycles per second the interference zones extend to a distance of 26 cms. from the transmitter. For frequencies up to about 80,000 cycles the pendulum will, therefore, be beyond the range covered by these zones but for higher frequencies the interference zones will extend to distances greater than 32 cms. and the pendulum will have to be moved. The change in position of the pendulum is indicated in table LII and also in figure 31.

TABLE LII

Steel Plates

Thickness of front plate	-	4.14	cms.
" " back "	-	4.22	"
" " quartz "	-	0.62	"
Total thickness		8.98	cms.

Voltage on Transmitter - 3000 volts
Energy Density calculated from equation (6) of section 11. -- $\frac{.329}{K}$ ergs per cu. cm.

(K to be determined from figure 34)
Thickness of pendulum vane - 0.254 cms.
Amplitude of vibration - .226 cms.

TABLE LII (contin.)

Frequency of vibration in cycles per sec.	Pendulum readings in radians.	Energy density in ergs	Amplitude of Vibrat-	Remarks
25,000	0	0	0	
25,500	0.016	,.0048	0.62×10^{-6} cms.	K =.75
27,000	0.065	.0182	1.01	K =.775
27,300	0.016	.0045	0.56	K =.800
29,400	0	0	0	
31,700	0	0	0	
32,500	0.014	.0037	0.42	K =.85
34,000	0.014	.0037	0.40	K =.86
33,900	0.024	.0063	0.53	K =.86
34,800	0.049	.0126	0.73	K =.87
35,700 (~~53.10~~)	0	0	0	
37,100	0.019	.0045	0.41	K =.875
38,100	0.020	.0051	0.42	K =.885
39,000	0.033	.0088	0.54	K =.890
39,500 (~~46.5~~)	0.015	.0038	0.36	do.
40,000 (~~45.4~~)	0.015	.0037	0.34	K =.90
40,000	0.015	.0037	0.34	K =.90
41,500 (~~42.0~~)	0.017	.0042	0.35	K =.90
42,500 (~~40.0~~)	0.019	.0046	0.36	K =.915
44,800	0.016	.0039	0.32	K =.915
46,400 (~~35.0~~)	0.016	.0039	0.31	K =.915
47,700	0.037	.0089	0.45	K =.930
50,000	0.008	.0019	0.20	K =.935
51,700	0.054	.0129	0.50	K =.94
51,700	0.049	.0117	0.47	K =.94
57,600	0.069	.0163	0.50	K =.95
64,400	0.050	.0117	0.38	K =.965
70,400	0.121	.0280	0.54	K =.970
Moved Pendulum out to 123.5 cms.				
40,000 to 68,500	0	0	0	
79,000	0.182	.039	0.56	K =.98
84,100	0.522	.144	1.02	K =.98
86,700	0.068	.016	0.33	K =.985
92,300	1.042	.238	1.19	K =.99
94,500	10.58	2.40	3.70	do.
96,500	1.19	,270	1.22	"
104,000	0	0	0	"
116,200	0.185	.042	0.40	"
130,500	0.275	.063	0.43	"
135,900	0.069	.016	0.21	"
139,000	0.292	.066	0.42	"
142,000	0.337	.077	0.44	"
145,000	0.418	.095	0.48	"
146,000	0.641	.145	0.59	"
151,500	0.660	.150	0.58	"
159,500	0.400	.091	0.45	"
169,500	7.76	1.766	1.69	"
170,500	3.31	7.208	1.46	"
179,500	1.30	.295	0.68	"
198,500	0.45	.102	0.36	"
222,000	0.81	.184	0.44	"

TABLE LIII

Steel Plates

Thickness of front plate - 1.74 cms.
" " back " - 1.69 "
" " quartz - 0.62 "
Total Thickness 4.05 "

Distance from Transmitter - 61 cms.
Voltage on Transmitter -- 3000 volts
Inergy density from equation (6) section 11 (a) $= \frac{0.1675}{K}\,\theta$ ergs per cu. cm.

(K to be determined from figure 34)
Thickness of pendulum vane -- 0.254 cms.
Amplitude of vibration - .226

Frequency of vibrations in cycles per sec.	Pendulum readings in radians	Energy density in ergs	Amplitude of vibration.	Remarks
46,400	.026	.041	0.99×10^{-6} cm	K=.920
49,500	1.54	.279	2.42	.935
50,000	3.45	.625	3.57	do.
50,300	2.44	.442	2.99	"
51,000	1.85	.330	2.55	.940
51,700	1.02	.182	1.87	do.
52,600	0.39	.069	1.05	.945
56,800	0.59	.103	1.28	.950
61,000	0.31	.054	0.80	.960
64,100	0.08	.014	0.42	.965

TABLE LIV

Marble Plates

Thickness of front plate - 1.69 cms.
" " back " - 1.74 "
" " quartz - 0.62 "
Total Thickness 4.05 "
Distance from Transmitter -- 61 cms.
Voltage on Transmitter -- 3000 volts
Energy Density $= \frac{.0221}{K}\,\theta$ ergs per cu. cm. (K to be determined from figure 34)
Thickness of pendulum vane -- 0.24 cms.
Amplitude of vibration = .226

Frequency of vibration in cycles per sec.	Pendulum readings in radians	Energy density in ergs	Amplitude of vibration	Remarks
49,400	.052	.0012	0.16×10^{-6} cms	K=.935
55,500	.09	.0021	0.19	.950
61,000	.25	.0057	0.28	.960
67,200	.16	.0036	0.20	.970
71,500	.21	.0047	0.22	.975

TABLE LIV (contin.)

Frequency of Vibration in cycles per sec.	Pendulum readings in radians	Energy density in ergs	Amplitude of vibration	Remarks
75,000	.12	.0027	0.16×10^{-6}	K = .975
78,000	.29	.0066	0.24	do.
78,200	.71	.0161	0.37	"
81,000	.66	.0150	0.34	.980
88,000	.42	.0094	0.25	.985
100,000	.09	.0020	0.03	.99
107,000	.02	.0004	0.01	do.

TABLE LV

Lead Plates

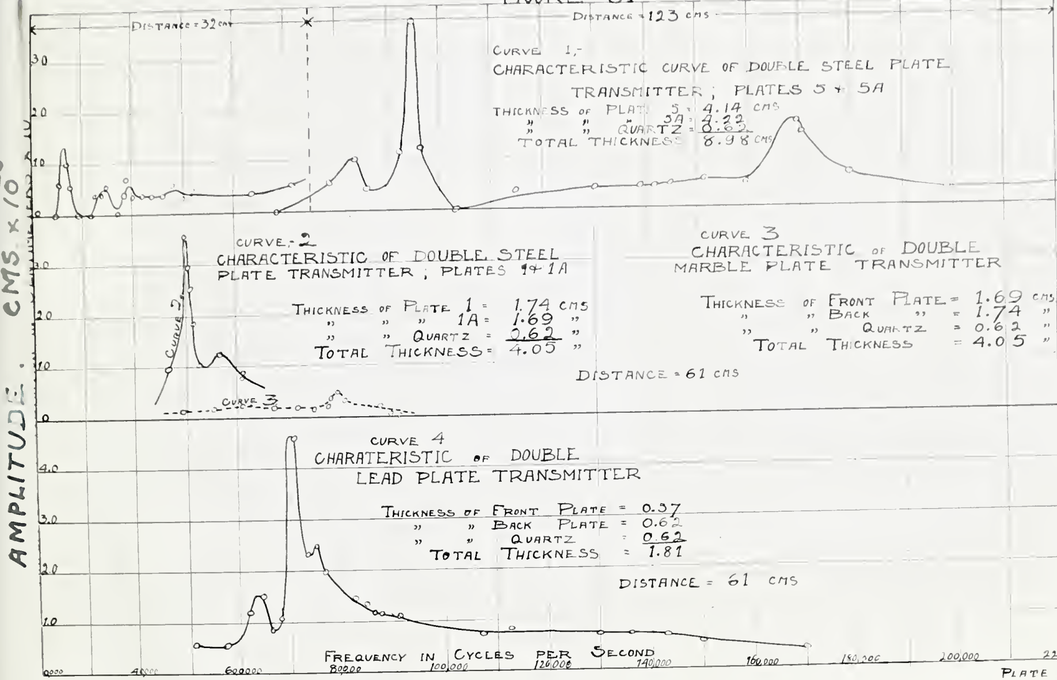
Thickness of front plate - 0.57 cms.
" " back " - 0.60 "
" " quartz - 0.62 "
Total thickness 1.79 "

Distance from Transmitter -- 61 cms.
Voltage on Transmitter -- 3000 volts
Energy density -- $\frac{.0237}{K}\theta$ ergs per cu. cm. (K to be determined from fig. 34)

Thickness of pendulum vane -- 0.254 cms.
Amplitude of vibration -- .226

Frequency of vibration in cycles per sec.	Pendulum readings in radians	Energy density in ergs	Amplitude of vibration	Remarks
51,000	0.07	.018	0.60×10^{-6} cms	K = .940
57,000	0.07	.018	0.54	K = .950
61,700	0.44	.108	1.20	K = .96
64,500	0.78	.191	1.53	K = .965
66,000	0.31	.062	0.85	K = .97
68,200	2.32	.567	1.10	do.
71,000	8.60	2.09	4.60	K " = .975
73,500	2.32	.563	2.31	do.
75,000	2.82	.579	2.48	"
76,500	1.97	.478	2.04	"
82,200	1.24	.299	1.50	K = .980
84,600	1.09	.265	1.57	do.
86,600	0.87	.209	1.19	K = .985
88,300	0.87	.208	1.17	do.
91,000	0.90	.215	1.18	K = .990
108,000	0.54	.129	0.75	do.
112,500	0.71	.176	0.84	"

FIGURE 51



AMPLITUDE . CMS × 10⁻⁶
DISTANCE = 32 CMS
DISTANCE = 123 CMS
CURVE 1,-
CHARACTERISTIC CURVE OF DOUBLE STEEL PLATE TRANSMITTER ; PLATES 5 + 5A
THICKNESS OF PLATE 5 = 4.14 CMS
" " " 5A = 4.22
" " QUARTZ = 0.62
TOTAL THICKNESS 8.98 CMS
CURVE - 2
CHARACTERISTIC OF DOUBLE STEEL PLATE TRANSMITTER ; PLATES 1 + 1A
THICKNESS OF PLATE 1 = 1.74 CMS
" " " 1A = 1.69 "
" " QUARTZ = 0.62 "
TOTAL THICKNESS = 4.05 "
CURVE 3
CHARACTERISTIC OF DOUBLE MARBLE PLATE TRANSMITTER
THICKNESS OF FRONT PLATE = 1.69 CMS
" " BACK " = 1.74 "
" " QUARTZ = 0.62 "
TOTAL THICKNESS = 4.05 "
DISTANCE = 61 CMS
CURVE 2
CURVE 3
CURVE 4
CHARATERISTIC OF DOUBLE LEAD PLATE TRANSMITTER
THICKNESS OF FRONT PLATE = 0.37
" " BACK PLATE = 0.62
" " QUARTZ = 0.62
TOTAL THICKNESS = 1.81
DISTANCE = 61 CMS
FREQUENCY IN CYCLES PER SECOND
PLATE

(b) Discussion of Results

Referring to curve 1, figure 51, we see that well marked "peaks" of energy emission occurred at frequencies of 26,000 cycles per second, 95,000 cycles per second and 160,000 cycles per second. These frequencies correspond to wave-lengths in steel of 18.9 cms.; 5.36 cms. and 3.02 cms. The thickness of the front plate was 4.14 cms., that of the back plate was 4.22 cms., while the quartz was 0.62 cms. thick. If we assume as an approximation that the velocity of sound in the quartz plate at the centre of the instrument is the same as in steel, and that the reflections at the quartz-steel surfaces do not matter appreciably, then the total effective thickness of the instrument is 8.98 cms. The first resonance peak occurred, therefore, when the total thickness of the instrument was 0.95 half wave lengths, or approximately one half wave length; the second when the total thickness was 3.3 half wavelengths; and the third when the thickness was 5.95 or nearly six half wave-lengths.

The above results show, that the simple half wave-length rule will not hold throughout the whole range of possible frequencies. It is evident that internal reflections, loss of energy by viscosity and by lateral vibration, by affecting the natural period of oscillation, will introduce considerable modifications when their effects are known more definitely.

It will be noticed that in nearly all the characteristic curves of both single and double plate instruments, there are secondary or minor peaks, always accompanying the main resonant peaks, usually one on each side of it. Most likely these are due to multiple internal reflections, as might be expected; but how they modify the frequency corresponding to exact resonance cannot yet be told. It is interesting to note, in this connection, that the second resonant peak shown in curve 1 of figure 51 is considerably higher than either the fundamental or the third resonant peaks; it has also been indicated that this peak occurred when the thickness of the instrument was 3.3 half wave-lengths which shows considerable dis-

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ok-
rep-

ancy with the half wave-length rule. Apparantly, at this frequency the instrument was emitting considerably more energy than at either the fundamental or the third resonant frequency.

Curve 1 figure 51 also indicates that there are no resonant peaks corresponding to frequencies at which the thickness of the transmitter would be equivalent to two or four half wave-lengths, that is at 57,000 cycles per second and 114,000 cycles per second. It has already been pointed out that resonance points cannot occur when the thickness of the instrument is an even number of half wave-lengths, because of the restriction that an antinode of pressure and node of amplitude must occur at the centre of the instrument. There does appear to be a resonance peak at six half wave-lengths. At this frequency, however, the effect of viscosity and lateral vibration must be considerable and, therefore, it is possible that what appears to be a peak at six half-wave lengths actually corresponds to either five or seven half-wave lengths, the position being modified by viscosity and other effects the magnitude of which ~~is~~ are, at present, unknown.

The remainder of the double plate transmitters were not investigated in as great detail as was the one considered above. Only the fundamental resonant frequencies of these instruments have been located. In curve 2, the characteristic of the second steel plate instrument is given, the thickness of the front and back plates being 1.74 and 1.69 cms., respectively. Resonance occurs at a frequency of 50,000 cycles per second which corresponds to a wave-length in steel of 10.2 cms. The total thickness of the instrument is 4.08 cms. which corresponds to 0.8 half wave-lengths. This value is decidedly less than one half wave-length but in the instrument under consideration the thickness of the quartz is ~~comparative~~ COMPARABLE with the thickness of the steel plates and therefore the assumption

that the quartz plate is equivalent to an equal thickness of steel is less justified. All the indications to date are that the velocity of sound in the direction of the electric axis of quartz is decidedly not the same as the velocity of sound in steel. If this is the case 0.62 cms of quartz may correspond to a considerably greater thickness of steel and the effective thickness of the instrument might thus be larger than the value quoted.

Curve 3 refers to the double marble transmitter. The radiations from this instrument are so small, comparatively, that it is of no practical importance as an ultra-sonic transmitter. From curve 3 it is evident ~~is seen~~ that materials whose physical properties are similar to those of non-metallic substances like marble will not make satisfactory plates for an ultra-sonic transmitter.

Curve 4 is the characteristic of the double lead plate transmitter. The thickness of the plates in this instrument were 0.57 and 0.60 cms, i.e. they were of about the same thickness as the quartz plates, viz. 0.62 cms. With the quartz plate the total thickness of the instrument was 1.79 cms. Resonance occurred at a frequency of 71,000 cycles per second. If we take the velocity of sound in lead as 2.1 x 10^5 cms. per second, this frequency of 71,000 cycles per second would correspond to a wave-length in lead of 2.96 cms. or a half wave-length of 1.48 cms. Therefore, the effective thickness of the present transmitter is less than 1.79 cms. This would be the case if the velocity of sound along the electric axis of the quartz were greater than the velocity of sound in lead. From a consideration of the resonance peaks of curves 2 and 4 it would appear that the velocity of sound along the electric axis of quartz is intermediate between the velocities in lead and steel. The point has to be settled later.

The chief point of interest in connection with the lead plate transmitter is the amplitude of vibration of the emitted energy. Comparing the amplitude of vibration of the fundamental resonant peaks in curves 1, 2 and 4, we see that the

amplitude of vibration of the lead instrument, ~~notwithstanding the fact that the frequency is higher~~, is 1.5 times the corresponding amplitude in the steel instrument with plates 1.7 cms. thick, and three times the corresponding amplitude for the instrument with plates 4.2 cms. thick. An energy loss due to viscosity and lateral vibrations occurs in the metal plates and the viscosity loss increases as the thickness of the plate increases.

Considering the low elasticity and high viscosity, relatively, of lead as compared with steel it might be expected that the vibrations in lead would be considerably damped and in consequence the amplitude of the emitted energy considerably less than in the case of steel instruments. But the low ~~viscosity~~ velocity of sound in lead, as compared with that in other metals, requires that comparatively thin plates of lead be used to obtain any specified wave-length. The results here would suggest that owing to the decrease in the thickness of the metal plate transmitter the viscosity and lateral losses in a lead plate transmitter may be smaller than such losses in an instrument of any other metal designed for the same resonant wave length. In actual practice it has been found that a lead transmitter of type C is considerably lighter and thinner than a steel one designed for the same resonant frequency. The decrease in the thickness of plate more than compensates for the increase in density of the lead.

Another possible cause of the comparitively large amplitude yielded by a lead plate instrument is probably in the fact that the lead gets a better grip on the quartz, through the cement, and the cemented joint is stronger than in the case of steel. The compressions and tensions of the wave in the metal must be conveyed across these cemented joints, and therefore the strength and permanency of the joint is a matter of great importance.

One outstanding feature of the double plate instruments is their extremely sharp resonant peaks. In fact with a single plate transmitter the resonant peak is quite sharp but with a double plate instrument the sharpness is accentuated.

See Fig. 49. This figure represents the characteristic curve of the 4.2 cm. double steel instrument, and is based on energy densities instead of amplitudes of vibration. It represents the conditions met with in practice. The figure shows that the total width of the resonant peak is only 3,000 or 4,000 cycles. At some points a frequency fluctuation of only 500 cycles, or less than 0.5 per cent would be sufficient to decrease the energy emission by about 60 per cent. It was found, when operating these instruments, that sometimes when an observer approached the electrical generating circuit the capacity of his body was sufficient to throw the frequency of vibration off the resonant value and thereby diminish the energy radiations by 50 per cent or more. The resonant peaks of these instruments must be broadened very considerably to ensure steady and consistent operations, for it is very difficult to keep the frequency of vibration steadily at the exact resonant value.

SECTION 18.

Multi-layer Quartz Transmitter. Type D.

a. Experimental procedure.

The fourth type of instrument investigated was one in which the metal plates were reduced to a minimum. The requisite thickness for resonance was obtained by piling up layers of quartz. A thin steel back-plate was used to give the necessary rigidity to the instrument, during construction. On this steel plate quartz mosaics, similar to the one used in the instruments previously considered, were piled one on top of the other. Each layer of quartz was separated from the adjacent ones by thin sheets of copper .046 cms. thick which served as the electrode of the instrument. All the air was expelled from the interstices of the quartz mosaic by embedding each piece of quartz in a hot mixture of wax and resin. When constructing the transmitter

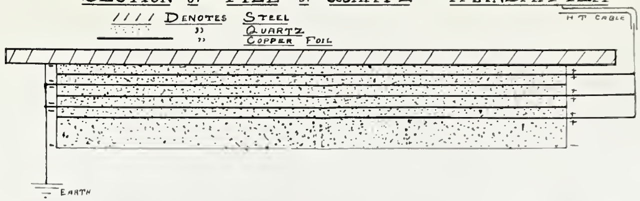
care had to be taken to make the direction of the piezo-electric distortion the same in each layer of quartz. This was done, as indocated in the introduction to Part 111, by placing the positive faces of two adjacent layers of quartz next to the same metal sheet, and similarly for the negative faces. A section of the pile of plates is sketched in Fig. 52, and the electrical connections are shown.

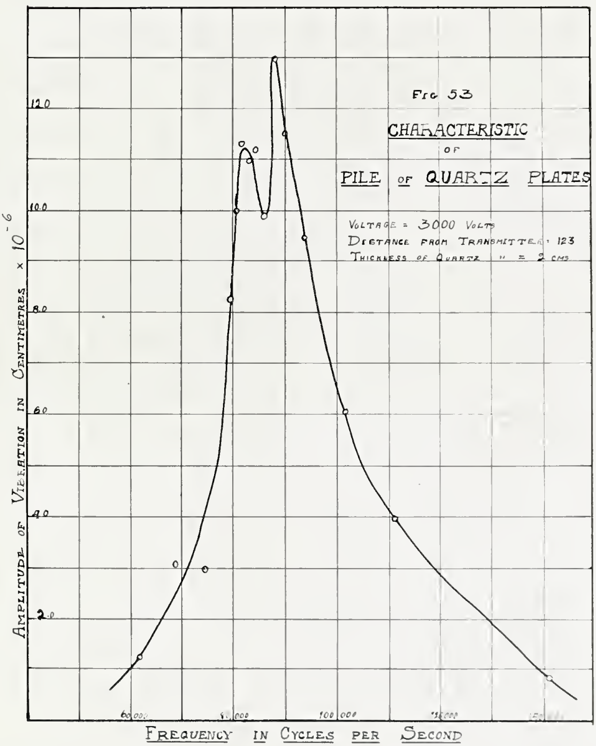
In the present instrument six layers of quartz were used. Unfortunately it was impossible to have these all of the same thickness, as the available supply of quartz was very limited and use had to be made of the few quartz mosaics on hand. The thickest layer consisted of the same quartz disc as was used in the earlier transmitters, the remaining layers being much thinner than this one. The total thickness of the pile was approximately two centimetres.

The characteristic curve of the multi-layer quartz instrument was obtained by the method described in section 15. In the present case, however, the energy emissions were very much greater than those obtained before. To obtain suitable pendulum readings it was necessary to operate the instrument at a voltage of 1,000 instead of 3,000 volts, and then, in order to make comparisons with the previous transmitters, to correct these readings to correspond to a voltage of 3,000. The data obtained is quoted in Table LV1 and the characteristic curve is plotted in figure 53. Readings have been obtained down to a frequency of 18,000 cycles per second but only the readings for frequencies above 40,000 cycles have been quoted. Below this frequency the energy emission was very small in comparison with that at the resonant peak. At low frequencies; about 20,000 cycles per second, a number of small peaks were obtained, due no doubt to the fact that the upper harmonics of the electrical circuit coincided with the resonant frequency of the transmitter.

Figure 52

SECTION OF PILE OF QUARTZ TRANSMITTER





PILE OF QUARTZ PLATES.

TABLE LVI

Corrected to Voltage 3,000.
Approximate thickness of Instrument 2 cms.
Distance from Transmitter 123 cms.

Energy Density $\frac{.225}{K} \theta$ ERGS PER CUBIC CENTIMETRE.

(K to be determined from Fig. 34)
Thickness of Pendulum Vane .254 cms.
Amplitude of Vibration $= .226 \sqrt{\frac{E_0}{n^2}}$ CENTIMETRES.

Frequency in cycles per sec.	Pendulum Reading in Radians.	Energy Density in Ergs per cu.cm.	Amplitude of Vibration.	Remarks.
61,700	0.47	0.11	1.22 x 10^{-6} cms.	K = .96
68,100	3.70	0.85	3.06 "	.97
74,100	3.85	0.89	2.98 "	.975
79,000	36.0	8.3	8.25 "	.98
80,000	55.0	12.6	10.0 "	"
81,000	71.0	16.3	11.3 "	"
82,600	70.0	16.1	11.0 "	"
84,500	76.1	17.4	11.2 "	"
85,500	61.1	14.0	9.9 "	"
87,500	110.0	25.2	13.0 "	"
87,500	110.0	25.2	13.0 "	"
89,500	95.5	20.1	11.5 "	.99
93,500	67.0	15.4	9.5 "	"
101,500	32.5	7.43	6.1 "	"
111,000	16.8	3.85	4.0 "	"
141,000	1.4	0.32	0.9 "	"

b. Discussion

Comparing the maximum amplitude shown in Fig. 53 with the maximum amplitude obtained from metal plate transmitters we see immediately that the multi-layer quartz instrument radiated far more energy than any instrument previously investigated. With the metal plate transmitters the amplitude of vibration in water was between 3 and 4 x 10^{-6} centimetres at their maximum values, whereas with the multi-layer quartz transmitter it was 13 x 10^{-6} centimetres and the energy emission, other things being equal is proportional to the square of the amplitude. For example the double lead plate instrument considered in section 18 had

a resonant frequency of 70,000 cycles per second, which is not far from the resonant frequency of the multi-layer instrument, viz; 87,500 cycles per second. The former had a resonant amplitude of 4.6×10^{-6} cms. Therefore the quartz transmitter produced an amplitude of vibration 2.7 times as great as that produced by the lead instrument and so radiated about 8 times as much energy.

In the multi-layer transmitter six layers of quartz were used, each of area equal to the active area in the former metal-plate instrument. Therefore the energy emitted should be increased six times for the same applied voltage. Also in the multi-layer transmitter the average thickness of each quartz layer was only half that of the quartz in the metal plate instruments, and therefore in the former the intensity of the electro-static field across the quartz for a given voltage, and consequently the energy emission, would be proportionally greater. Increasing the effective radiating surfaces by piling quartz plates one behind the other was preferable to increasing the actual area of the face of the instrument for two reasons: First, an increase in the radius of the radiating surface would decrease the angle of the ultrasonic beam which effect is frequently not desired, Second, increasing the thickness of the quartz pile makes it possible to obtain a resonant peak within the desirable range of frequency. This would not be possible if a single quartz disc were used for under these conditions the resonant frequency would be far too high for practical purposes.

A consideration of the above factors would indicate an energy emission from the quartz pile somewhat greater than can be realised in practice; for there must be considerable energy loss in the pile due to viscous damping in the cement between layers, and multiple-reflection at the many interior inter-faces. In spite of these losses the multi-layer instrument radiates far more energy than any other type as yet considered.

The radiating power of this transmitter could be obtained by the method outlined in section 11,c. These calculations will be carried out at a later date and will no doubt show a radiating power considerably greater than that of the instrument first investigated.

Referring to the characteristic curve shown in Fig. 53 it is noticeable that the resonant peak of the multi-layer ~~instrument~~ quartz transmitter is much broader than that of the double metal plate one shown in Fig, 51. This broadening is greatly to be desired as it tends to steady the operating conditions. It has been mentioned before that it is almost impossible to operate at a constant energy emission with the double plate transmitter on account of the frequency fluctuations which occur. In these instruments a very small change in frequency near the peak produces a large increase or decrease in the energy emitted. The resonant peak of the multi-layer transmitter is not, however, appreciably broader than that of the single-plate instrument as shown in Fig. 50. It should also be moticed that in all characteristic curves including the multi-layer one, secondary peaks accompany the principal resonant ones. These, no doubt are due to multiple internal reflections.

From the resonant frequency it is possible to estimate the velocity of ultra-sonic vibrations in the direction of the electric axis in the quartz pile, for it has already been shown in sections 15 to 17 that resonance occurs when the thickness of the instrument is approximately one half wave-length. No accurate calculations can be made until the effect of viscous damping, lateral vibrations, reflections at the interior inter-faces, etc., have been definitely established, but the indications are that ultrasonic velocity along the electric axis in quartz is in the neighborhood of 3 or 4 x 10^5 centimetres per second.

The experiments in this last section have shown that it is possible by making a pile of quartz plates, to obtain a transmitter of high

radiating power with a natural period of mechanical vibration convenient for ultra-sonic energy emission. It may be desirable to reduce the frequency of resonance still lower. This may be done by increasing the number of plates in the pile and thereby increasing its thickness. If sufficient quartz is not available the resonance frequency can still be lowered by loading the pile at the front and back with plates of lead, steel or other metal. By throwing the whole pile, so formed, into a resonant vibration a powerful radiation could be obtained. Such a pile loaded with lead plates is now being investigated.

Conclusion.

In summarizing the work covered in Part 111 the following conclusions are arrived at:

1. The resonant frequency of a single metal plate transmitter of type A may be obtained, but only approximately, by considering resonance to occur when the thickness of the backplate is one half wavelength. A convenient analogy is to consider the back-plate to be an open organ pipe and the vibrating quartz plate to be an exciting source. But this consideration is only valid when the quartz is thin in comparison with the thickness of the metal plate.

Similarly, as a first approximation, the double plate instrument of type C may be considered to have a fundamental note with half wavelength equal to the total thickness of the plate, but this simple rule will not hold so well for the upper resonant points at higher frequencies.

2. The most satisfactory material to make transmitter plates is lead for two reasons; First, comparatively thin sheets may be used and Second, the cement joining the metal plate to the quartz adheres to lead better than to other metals. Thus a stronger and more permanent joint results when lead plates are used.

3. A multi-layer instrument of type D emits far more energy than does any other type of instrument so far investigated and is to be preferred, if sufficient quartz is available. The resonant peak in this case is decidedly broader than that of a double metal plate instrument and appears to give more stable operation.

These investigations are being continued and further information on the subject will be reported at a later date.